On the transition from collisionless to collisional magnetohydrodynamics

R. J. Strangeway and J. Raeder

Institute of Geophysics and Planetary Physics, University of California, Los Angeles

Abstract. Magnetosphere-ionosphere coupling entails the interaction of two quite different plasmas. The magnetosphere is to a large extent a collisionless magnetohydrodynamic (MHD) fluid, while the ionosphere is strongly collisional. As such the relationship between the electric currents and electric field appear to be fundamentally different in the two regimes. For the magnetosphere the currents are determined by the forces within the plasma, while in the ionosphere the current and electric field are related through an anisotropic Ohm's law. Here we explore the transition between these two regimes and show that there is a clear ordering of the governing equations, with a single "collisional Ohm's law" that contains both the collision-frequency-dependent Pedersen and Hall conductivities and collisionless MHD terms, without making any prior assumptions concerning the ordering of the collision frequencies. The generalized Ohm's law of MHD is also present, but this equation reduces to the statement that within the ionosphere the magnetic field is "frozen" to the electron fluid, unless electron collision frequencies become comparable to the electron gyrofrequency. It is the freezing-in of the electron fluid that leads to the direct equivalence of mechanical and electromagnetic loads. This equivalence of loads also indicates that Poynting flux traveling upward out of the ionosphere can only occur if there is a convergence of horizontal Poynting flux in excess of the ionospheric Joule dissipation.

1. Introduction

An important question for the Earth's magnetosphere concerns the interplay between the magnetosphere and ionosphere. This magnetosphere-ionosphere coupling involves both electromagnetic and mechanical coupling. Electromagnetic coupling addresses the generation and dissipation of electric currents, while the mechanical coupling addresses how the magnetosphere imposes its flow patterns onto the ionosphere, which is subject to frictional drag through collisions with the neutral atmosphere. As a general rule, there is thought to be a strong equivalence between the two types of coupling, with the magnetosphere usually acting as a source of both mechanical and electromagnetic energy for the ionosphere. Occasionally, the ionosphere can drive the magnetosphere, by neutral winds, for example [*Deng et al.*, 1991], but in this case the ionosphere is both a mechanical driver and an electromagnetic generator.

In several instances, however, it appears that the ionosphere could drive the magnetosphere. A classic example is given by *Siscoe* [1982], who discusses the change in the region 2 current system following an increase in the applied potential of the region 1 system. He assumes that the two current systems are only connected through ionospheric Pederson currents, and consequently, "energy is transferred from the region 1 to the region 2 Birkeland current system via, effectively, an ionospheric battery" [*Siscoe*, 1982, p. 5126].

High-latitude radar and magnetometer observations also indicate that the polar cap convection changes quickly over the entire polar cap following a change in the interplanetary magnetic field (IMF). This has been argued as evidence that the "ionospheric electric field controls tail dynamics" [*Ridley et al.*, 1999, p. 4396; *Lockwood and Cowley*, 1999].

More recently, *Song et al.* [1999, 2000] found a region 1 sense current system deep within the magnetosphere for northward IMF using a global magnetohydrodynamic (MHD) simulation with a conducting ionosphere. They argued that this current system corresponded to the ionosphere acting as a mechanical driver of magnetospheric convection while simultaneously acting as an electromagnetic load.

All these examples appear to contradict the assertion that there is a direct equivalence between electromagnetic and mechanical loads. Our purpose here is therefore twofold. First, we wish to verify that the transition from the ideal MHD regime to the collisional regime is smooth. P. Song (personal communication, 2000) has suggested that the breakdown in equivalence of loads may occur because the magnetosphere and ionosphere are not governed by the same set of equations. The magnetosphere is generally thought to be governed by ideal MHD, although clearly MHD breaks down in certain regions, such as where reconnection is occurring, or where parallel electric fields are present. The ionosphere, on the other hand, is strongly collisional, and the current and electric field are directly related through an Ohm's law that includes Pedersen, Hall, and parallel conductivities. Intuition suggests that the transition is smooth, but there is always the danger of making assumptions concerning the ordering of terms, only to find that two assumed large terms cancel, while a smaller term which should be present had been dropped earlier.

Second, having verified that the transition from ideal MHD to strongly collisional MHD is indeed well ordered, we wish to elucidate the circumstances under which the ionosphere may appear to drive the magnetosphere. At its simplest, we will argue that this reduces to determining if Poynting flux flows into or out of the ionosphere.

While this could be viewed as a largely pedagogical exercise, fundamental insights into magnetosphere-ionosphere coupling are possible. In particular, the principle of “frozen-in” flow, generally thought to apply in the MHD regime, also applies for the ionosphere, provided it is realized that the magnetic field is frozen to the electron fluid, not the ion fluid. (An aside may be in order here: one often reads of “frozen-in electrons,” and we shall use the same phrase here, but it should be remembered that it is the magnetic field that is frozen to the electron fluid.) We emphasize that the concept of frozen-in electrons is well known. *Scudder et al.* [1999], for example, discuss the generalized Walén relation for the electron fluid. Our discussion here is another case where the magnetic field convects with the electron fluid rather than the ion fluid. It is this freezing-in with respect to the electron fluid that results in the equivalence of mechanical and electromagnetic loads and that also allows us to discuss the flow of magnetic energy through Poynting flux.

In the next section we will present the derivation of collisional MHD in a frame defined by the neutral atmosphere, showing that with a suitable gathering of terms the equations can be made compact, allowing for the ordering of the terms to be easily verified without making any prior assumptions. We are using the appellation “collisional MHD” since we are still using fluid equations. In addition, we are mainly concerned with the momentum equations of MHD, not the higher order moments, such as the energy equation. The third section will briefly discuss the extension of the formalism to other frames of reference, as well as the inclusion of additional forces and anomalous collisions. In the fourth section we will briefly reiterate the point that the ordering of the terms in collisional MHD holds no surprises. We will also make the point that in collisional MHD there is an Ohm’s law that relates the current to the electric field, which comes from a combination of the ion and electron momentum equations. The generalized Ohm’s law does not fulfill the role of relating the current to the electric field even on including collisions, since it is essentially the electron momentum equation, and therefore relates the electron flow velocity to the electric field. Rather, it demonstrates the freezing-in of the magnetic field with respect to the electron fluid. The frozen-in condition for electrons allows us to prove the direct equivalence between electromagnetic and mechanical loads, even in the collisional regime. Concluding remarks are given in the final section.

2. Basic Derivation

In order to derive an MHD formalism that includes collisions we shall assume that both electrons and ions can be treated as fluids. As we allude to later, nonfluid processes, such as wave-particle interactions could be included if the effects of such processes can be parameterized in terms of fluid properties.

The electron momentum equation is

$$m_e \frac{d\mathbf{U}'_e}{dt} = -e\left(\mathbf{E}' + \mathbf{U}'_e \times \mathbf{B}\right) - \frac{\nabla P_e}{n} - m_e \nu_{en} \mathbf{U}'_e + m_e \nu_{ei} \frac{\mathbf{j}}{ne}, \quad (1)$$

while for ions

$$m_i \frac{D\mathbf{U}'_i}{Dt} = e\left(\mathbf{E}' + \mathbf{U}'_i \times \mathbf{B}\right) - \frac{\nabla P_i}{n} - m_i \nu_{in} \mathbf{U}'_i - m_e \nu_{ei} \frac{\mathbf{j}}{ne}, \quad (2)$$

with the following definitions

m_e	electron mass;
m_i	ion mass;
$\mathbf{U}'_e$	electron flow velocity with respect to neutrals;
$\mathbf{U}'_i$	ion flow velocity with respect to neutrals;
$d/dt = \mathbf{U}'_e \bullet \nabla + \partial/\partial t'$	total time derivative operator for electron fluid;
$D/Dt = \mathbf{U}'_i \bullet \nabla + \partial/\partial t'$	total time derivative operator for ion fluid;
e	magnitude of the electron charge;
$\mathbf{E}'$	electric field in the frame of the neutrals;
$\mathbf{B}$	magnetic field;
P_e	electron thermal pressure;
P_i	ion thermal pressure;
n	number density;
ν_{en}	electron-neutral collision frequency;
ν_{in}	ion-neutral collision frequency;
ν_{ei}	electron-ion (Coulomb) collision frequency;
$\mathbf{j} = ne(\mathbf{U}'_i - \mathbf{U}'_e)$	current density

[e. g., *Barakat and Schunk*, 1982].

In writing these equations we have assumed that both species have the same number density n (i.e., space charge is ignored, as in the MHD approximation). Furthermore, since momentum is conserved for collisions between ions and electrons, $m_e \nu_{ei} = m_i \nu_{ie}$, where ν_{ie} is the ion-electron collision frequency. We have also used different total time derivative operators for each species to allow for the possibility of large current densities, as occur deep in the ionosphere where ion-neutral collisions strongly retard the ion motion. We should also emphasize that the velocities and electric field are determined in the frame moving with the neutrals, as indicated by being primed. Unprimed variables are invariant under a Galilean transformation. The current density and total time derivative operators are also frame invariant, even though they have been defined in the frame of the neutrals.

It should be noted that we have ignored many of the issues involved in obtaining the electron and ion momentum equations in the form given above. In particular, we have reduced the collisional terms to their simplest form, only involving first-order velocity moments of the phase space distribution and velocity-independent collision frequencies. *Green* [1959], for example, argues that this can only be done if there are no temperature gradients or if thermoelectric effects are unimportant. Such a discussion is well beyond the scope of this paper, but we expect any corrections along these lines to be small.

To further simplify these equations, we will define several force terms

$$\mathbf{F}_e = -\nabla P_e - n m_e \frac{d\mathbf{U}'_e}{dt}, \quad (3)$$

$$\mathbf{F}_i = -\nabla P_i - n m_i \frac{D\mathbf{U}'_i}{Dt}, \quad (4)$$

$$\mathbf{F} = \mathbf{F}_e + \mathbf{F}_i, \quad (5)$$

and a modified electric field

$$\tilde{\mathbf{E}}' = \mathbf{E}' - \frac{\mathbf{F}_e}{ne} - \frac{m_e \nu_{ei}}{ne^2} \mathbf{j}. \quad (6)$$

The force terms defined above are also invariant under a Galilean transformation, since the velocities in different frames differ by a constant. Furthermore, $\mathbf{F}_e$ and $\mathbf{F}_i$ are strictly speaking force densities (N/m^3). We could as easily have defined them in units of force, but it is the normal convention in MHD to use force densities, even though they are referred to as forces (e.g., the "$\mathbf{j}\times\mathbf{B}$ force"). We will use the same shorthand here. We only need to remember the distinction when we wish to include additional forces. The force definitions include the rate of change of momentum, which may be counterintuitive. On the one hand, the spatial derivative can be interpreted as a gradient in the dynamic pressure (plus vorticity terms), but on the other hand, we are treating the time rate of change of momentum as a force. Nevertheless, it is common to refer to inertial forces as generating currents [e.g., *Haerendel*, 1990], and including the inertial terms in $\mathbf{F}_e$ and $\mathbf{F}_i$ is not a problem as long as it is remembered that force balance requires the sum of all the forces as defined to equal zero. The meaning of $\tilde{\mathbf{E}}'$ will become clear on rewriting the electron and ion momentum equations:

$$m_e \nu_{en} \mathbf{U}_e' = -e\left(\tilde{\mathbf{E}}' + \mathbf{U}_e' \times \mathbf{B}\right), \quad (7)$$

$$m_i \nu_{in} \mathbf{U}_i' = e\left(\tilde{\mathbf{E}}' + \frac{\mathbf{F}}{ne} + \mathbf{U}_i' \times \mathbf{B}\right). \quad (8)$$

Equation (7) is a modified form of the generalized Ohm's law of MHD. When electron-neutral collisions are unimportant, we have $\tilde{\mathbf{E}}' + \mathbf{U}_e' \times \mathbf{B} = 0$, which on expanding the terms becomes

$$-\frac{m_e}{e}\frac{d\mathbf{U}_e'}{dt} = \mathbf{E}' + \mathbf{U}_i' \times \mathbf{B} - \frac{\mathbf{j}\times\mathbf{B}}{ne} + \frac{\nabla P_e}{ne} - \frac{m_e \nu_{ei}}{ne^2}\mathbf{j}. \quad (9)$$

This is the MHD generalized Ohm's law, except for the modification that we use $-d\mathbf{U}_e'/dt$, rather than $(1/ne)\partial\mathbf{j}/\partial t$. The latter is often used in the classical generalized Ohm's law because terms quadratic in $\mathbf{U}_e'$ are assumed to be small, and $|\partial\mathbf{U}_i'/\partial t| \ll |\partial\mathbf{U}_e'/\partial t|$ because of the difference in masses [see, e.g., *Boyd and Sanderson*, 1969]. Equation (9) has not yet made any such simplifying (albeit reasonable) assumptions. We also note that, strictly speaking, keeping only the $\partial\mathbf{j}/\partial t$ term breaks the Galilean invariance of the generalized Ohm's law.

On adding (7) and (8) we find

$$n\left(m_i \nu_{in} \mathbf{U}_i' + m_e \nu_{en} \mathbf{U}_e'\right) = \mathbf{F} + \mathbf{j}\times\mathbf{B}. \quad (10)$$

Equation (10) is the full momentum equation. The momentum equation written in this form also shows why the definitions in (3) – (6) were chosen. The force terms on the right-hand side have no collision terms; in particular, it is important to remember that there are no terms dependent on electron-ion collisions in (10), although such collisional terms are included in the ion and electron momentum equations separately (through the modified electric field $\tilde{\mathbf{E}}'$). This is of course a direct result of conservation of momentum for such collisions. In the absence of collisions with neutral particles, (10) becomes the standard MHD momentum equation $\mathbf{F} + \mathbf{j}\times\mathbf{B} = 0$.

The momentum equation (10) also shows a clear separation of the force terms. The left-hand side shows the frictional drag terms, while the first term on the right-hand side includes all the nonelectromagnetic forces. The last term is of course the $\mathbf{j}\times\mathbf{B}$ force. This is the force term through which the magnetic field can do work on the plasma. We will return to this point in our discussion.

One could proceed by using the modified generalized Ohm's law (7) and the momentum equation (10), using the definition of current density to replace the electron bulk velocity. Instead, we will solve for the perpendicular components of the species bulk flow velocities separately.

$$\mathbf{U}_{e\perp}'\left(1 + \nu_{en}^2/\Omega_e^2\right) = \frac{\tilde{\mathbf{E}}_\perp' \times \mathbf{B}}{B^2} - \frac{\nu_{en}}{\Omega_e}\frac{\tilde{\mathbf{E}}_\perp'}{B}, \quad (11)$$

$$\mathbf{U}_{i\perp}'\left(1 + \nu_{in}^2/\Omega_i^2\right) = \frac{(\tilde{\mathbf{E}}_\perp' + \mathbf{F}_\perp/ne)\times\mathbf{B}}{B^2} + \frac{\nu_{in}}{\Omega_i}\frac{(\tilde{\mathbf{E}}_\perp' + \mathbf{F}_\perp/ne)}{B}, \quad (12)$$

where Ω_e and Ω_i are the electron and ion gyrofrequencies, respectively (eB/m_e and eB/m_i).

The classical definitions of ionospheric Pedersen and Hall conductivities are [e. g., *Luhmann*, 1995]

$$\sigma_p = \frac{ne}{B}\left(\frac{\nu_{en}/\Omega_e}{1 + \nu_{en}^2/\Omega_e^2} + \frac{\nu_{in}/\Omega_i}{1 + \nu_{in}^2/\Omega_i^2}\right) \quad (13)$$

$$\sigma_h = \frac{ne}{B}\left(\frac{1}{1 + \nu_{en}^2/\Omega_e^2} - \frac{1}{1 + \nu_{in}^2/\Omega_i^2}\right). \quad (14)$$

Both σ_p and σ_h are positive quantities, although formally σ_h could be negative. In general we expect $\nu_{in}/\nu_{en} \approx v_{Ti}/v_{Te}$, where v_{Ti} and v_{Te} are ion and electron thermal velocities, and hence $\nu_{in}/\nu_{en} \approx (m_e/m_i)^{1/2}$, especially in the ionosphere where $T_e \approx T_i$. Consequently, $r = (\nu_{en}/\Omega_e)/(\nu_{in}/\Omega_i) \approx (m_e/m_i)^{1/2}$ within the ionosphere. At higher altitudes, electron temperatures may be higher than ion temperatures, but generally $r \ll 1$, and the first term in parentheses on the right-hand side of (14) is always larger than the second. Thus σ_h as defined is positive. (See also *Kelley* [1989, chap. 2] for a discussion of the ordering of the collision frequencies.) With these definitions of σ_p and σ_h, (11) and (12) give

$$\mathbf{j}_\perp = \sigma_p \tilde{\mathbf{E}}_\perp' - \sigma_h \frac{\tilde{\mathbf{E}}_\perp' \times \mathbf{B}}{B} + \frac{\nu_{in}/\Omega_i \, \mathbf{F}_\perp/B}{1 + \nu_{in}^2/\Omega_i^2} + \frac{\mathbf{F}_\perp \times \mathbf{B}/B^2}{1 + \nu_{in}^2/\Omega_i^2}, \quad (15)$$

which we shall refer to as the collisional MHD Ohm's law.

For completeness, the parallel current density is obtained from the difference of the parallel components of (7) and (8), divided by $m_e \nu_{en}$ and $m_i \nu_{in}$, respectively,

$$j_\parallel\left(\nu_{en} + \nu_{ei}(1 + r)\right) = \frac{ne^2}{m_e}\left(E_\parallel(1 + r) - \frac{F_{e\parallel}}{ne} + r\frac{F_{i\parallel}}{ne}\right), \quad (16)$$

where r is as defined above. Note that we have expanded $\tilde{\mathbf{E}}'$ in (16), as this field includes $j_\parallel$ through the Coulomb collision term. Also, $E_\parallel$ and hence $j_\parallel$ are of course frame invariant.

Again, noting that $r \approx (m_e/m_i)^{1/2}$, and neglecting terms of this order,

$$j_\parallel\left(\nu_{en} + \nu_{ei}\right) = \frac{ne^2}{m_e}\left(E_\parallel - \frac{F_{e\parallel}}{ne}\right). \quad (17)$$

The parallel collisional conductivity therefore depends on the total electron collision frequency $\nu_{en} + \nu_{ei}$. For vanishing collision frequencies, and assuming electron inertia terms can be ignored, (17) gives the standard result $neE_\parallel = -(\nabla P_e)_\parallel$, that is, a parallel electric field is maintained by electron thermal pressure in the collisionless regime.

At this stage we shall concentrate on (15), since the perpendicular currents are important for magnetosphere-ionosphere coupling, although field-aligned currents clearly play a key role in coupling the perpendicular currents in the magnetosphere and ionosphere. It should also be noted that (17) generally requires modification because of anomolous resistivity, where such resistivity may be caused by wave-particle interactions, or other non-MHD processes such as “Knight resistivity” [*Knight*, 1973].

Equation (15) is useful in determining the relative importance of various terms, without having made a priori assumptions concerning the ordering of the different collision frequencies. One immediate consequence of (15) is that Coulomb collisions only enter through the modified electric field $\tilde{\mathbf{E}}'$ and therefore only become potentially relevant when ion- and electron-neutral collisions are themselves important. However, even then, the effect of Coulomb collisions is minor. This can be seen by noting that σ_p and σ_h are $\leq ne/B$. As a consequence, the correction to $\mathbf{j}_\perp$ associated with the Coulomb collision term in (6) is $\leq \nu_{ei}/\Omega_e$. This is a very small correction, $< 10^{-3}$ for typical ionospheric parameters. Thus for the terrestrial ionosphere and magnetosphere (where ν_{ei}/Ω_e is even smaller), the ordering of the terms in (15) is determined by the ion- and electron-neutral collision frequencies, to a very good approximation.

Before discussing the relative importance of the various terms in (15), we will address some remarks as to the difference between $\tilde{\mathbf{E}}'_\perp$ and $\mathbf{E}'_\perp$. We have already established that the Coulomb collision term is unimportant, so the primary difference is in the $\mathbf{F}_{e\perp}/ne$ term. In general, this is also small in comparison to the electric field. Electric fields are typically observed to be of order mV/m in the magnetosphere, and tens or even hundreds of mV/m in the ionosphere. As a consequence, we would require gradients in the energy of the electrons of order eV/km for the force term to be comparable to the electric field. In dimensionless units, electron pressure gradients become important when $(\rho_e/L)(v_{Te}/U_e) \approx O(1)$, where ρ_e is the thermal electron Larmor radius, and L is the pressure gradient scale length. Gradients of sufficiently small scale may exist in the magnetospheric plasma sheet, for example, especially when the plasma sheet becomes very thin, or in the magnetopause reconnection layer, but we do not expect such gradients to be present in the topside ionosphere. For completeness, we will still use $\tilde{\mathbf{E}}'_\perp$, although we could generally use $\mathbf{E}'_\perp$ without significantly altering our analysis.

Ordering the various terms in (15) by ν_{in}/Ω_i, we find

$$j_\perp B\left(1 + \nu_{in}^2/\Omega_i^2\right) : \nu_{in}/\Omega_i\left(\frac{1 + r\nu_{in}^2/\Omega_i^2}{1 + r^2\nu_{in}^2/\Omega_i^2}\right)ne\tilde{E}'_\perp$$

$$: \nu_{in}^2/\Omega_i^2\left(\frac{1}{1 + r^2\nu_{in}^2/\Omega_i^2}\right)ne\tilde{E}'_\perp : \nu_{in}/\Omega_i F_\perp : F_\perp \,.$$

In order, the terms correspond to the total perpendicular current density, the Pedersen current density, the Hall current density, a force-dependent term, and the current density due to $\mathbf{F}\times\mathbf{B}$. For vanishing ion-neutral collisions, only the last term remains on the right-hand side of (15), and we again have the classical MHD result, where the current is determined from $\mathbf{F}\times\mathbf{B}$.

As ν_{in}/Ω_i increases, corresponding to moving from higher (magnetospheric) altitudes to lower (ionospheric) altitudes, the other terms become more important. Assuming that $ne\tilde{E}'_\perp >> F_\perp$, the Pedersen current becomes important when $\nu_{in}/\Omega_i \approx F_\perp/ne\tilde{E}'_\perp$. This condition marks the transition from collisionless to collisional MHD, and more importantly, the transition to a plasma where $\mathbf{j}_\perp \bullet \tilde{\mathbf{E}}'_\perp > 0$ in the neutral frame. When $\nu_{in}/\Omega_i > F_\perp/ne\tilde{E}'_\perp$ the dot product of (15) with $\tilde{\mathbf{E}}'_\perp$ gives

$$\mathbf{j}_\perp \bullet \tilde{\mathbf{E}}'_\perp = \sigma_p \tilde{E}'^2_\perp + \text{smaller terms}\,, \qquad (18)$$

where the σ_h dependent term of course drops out. In the collisionless MHD regime there is no constraint on the sign of $\mathbf{j}_\perp \bullet \tilde{\mathbf{E}}'_\perp$.

When $\nu_{in}/\Omega_i \approx 1$, the Pedersen and Hall currents in (15) are comparable, while the Pedersen current again dominates for $\nu_{in}/\Omega_i >> 1$. In this case the Pedersen current is being carried by electrons, whereas the Pedersen current is carried by ions for lower collision frequencies (higher altitudes). A further comment may be in order here. In discussing the ordering of the terms we have not made any assumptions concerning the magnitude of the collision frequencies. For the electrons to carry the Pedersen current requires $\nu_{en}/\Omega_e \approx 1$. While this is technically possible, $\nu_{en}/\Omega_e << 1$ within the Earth’s E and F region ionosphere [*Kelley*, 1989].

To summarize our discussion so far, we have derived an Ohm’s law which encompasses both collisionless and collisional MHD, in (15), without making any prior assumptions concerning the ordering of the various terms. This Ohm’s law is not the generalized Ohm’s law of weakly collisional MHD (when only Coulomb collisions are included), indeed for vanishing ion- and electron-neutral collision frequencies, there is no direct electric field dependence in (15). This is of course another way of stating that in ideal MHD the electric field is related to the flow velocity, while the current is related to the forces in the plasma. On including collisions with neutrals, (7) takes over the role of the generalized Ohm’s law, while (15) takes the role of the momentum equation in determining the current density. While we refer to (15) as a collisional MHD Ohm’s law, it also reduces to the momentum equation in the collisionless limit. Besides explicitly containing Pedersen and Hall conductivities, (15) is useful in showing that there is a smooth transition from the collisionless to the collisonal regime with a clear hierarchy in the different terms contributing to the perpendicular current.

3. Extensions

The derivation given above made some initial assumptions. The first of these is that flow velocities and electric fields are specified in the frame of the neutrals. In addition, we assumed that the plasma pressure was isotropic with respect to the magnetic field direction, and no other nonelectromagnetic forces operated. With certain caveats, these assumptions can be relaxed, and the general form of (15) remains.

3.1 Different Plasma Frames

The first condition to relax is that the formalism be derived in the frame of the neutrals. Indeed, this constraint must be relaxed if we are to consider the effects of neutral winds, for example. Under a Galilean frame transformation the current density and magnetic field are invariant, although the electric field and velocity are not. Thus (15) should apply regardless of the frame of reference. If we assume that the neutrals are moving with a velocity $\mathbf{U}_n$ with respect to the new frame of reference, then transformation of the electric field is of course given by $\mathbf{E}' = \mathbf{E} + \mathbf{U}_n \times \mathbf{B}$, and the velocities transform as $\mathbf{U}' = \mathbf{U} - \mathbf{U}_n$, where unprimed variables are in the inertial frame. All the other terms in $\tilde{\mathbf{E}}'$ are invariant, and the Pedersen and Hall terms in (15) are therefore also invariant, provided the electric fields and velocities are transformed from the observer frame to the local neutral frame by the above transformations. As noted earlier, $\mathbf{F}_\perp$ is invariant under a Galilean transformation and the remaining terms in (15) are therefore invariant under a Galilean frame transformation. The current density given by (15) is indeed frame independent.

3.2 Additional Forces

It is clear that the force terms $\mathbf{F}_i$ and $\mathbf{F}_e$ can be expanded to include other forces, such as gravity and anisotropic pressure. The only complication may arise from forces which couple the parallel and perpendicular currents. Indeed, it should be noted that the advective derivatives in (3) and (4) could also couple parallel and perpendicular motions, corresponding to vorticity in the flows. In the light of the ordering scheme discussed above, however, it seems reasonable to assume that any forces due to parallel motion are minor in the collisional regime, and the perpendicular current is largely unaffected in this regime. Thus (15) can be extended to include additional force terms while preserving the ordering of terms.

3.3 Anomalous Collisions

Any anomalous collisions processes that result in the transfer of momentum between electrons and ions could be incorporated into the system of equations as a modification to the Coulomb collision frequency. As such, these processes could strongly modify the parallel current (see (17)) but would have no direct effect on perpendicular currents, unless the effective electron collision frequency becomes comparable to the electron gyrofrequency. Any other anomalous collision processes which do not conserve plasma momentum could be included in the neutral collision frequencies, thereby modifying the Hall and Pedersen conductivities. One complication of this approach, however, concerns the choice of reference frame, as the rate at which momentum is lost is determined by the velocity with respect to the scattering center. Instead of using the frame of the neutrals, we could reformulate the problem in the appropriate reference frame, including additional acceleration terms in $\mathbf{F}_i$ and $\mathbf{F}_e$ to account for the momentum gained by collisions with the moving neutrals. The hierarchy of terms in the revised (15) would be less clear, however, although the simplicity of the form of the equations would be retained.

4. Discussion

We have derived a collisional MHD formalism based on the fluid ion and electron momentum equations including collisions. Perhaps not surprisingly, we obtain an Ohm's law (equation (15)) that shows a clear separation of collision-dominated and MHD-dominated terms, although we made no a priori assumptions concerning the relative ordering of the collision frequencies.

In collisionless (or weakly collisional) MHD the ion and electron momentum equations are usually recast into a generalized Ohm's law and a force law, with the latter being used to relate the current density to the other (nonelectromagnetic) forces in the plasma, since the current density enters as a Lorentz ($\mathbf{j}\times\mathbf{B}$) force. It is the $\mathbf{j}\times\mathbf{B}$ force term which enables the magnetic field to do work on the plasma. The formalism presented here retains this structure, in (7) and (10), although we have also derived a "collisional Ohm's law" (equation (15)) to more clearly demonstrate the dependence of the current density on the electric field through the Pedersen and Hall conductivities, in addition to terms related to the nonelectromagnetic forces.

The distinction between this collisional Ohm's law and the generalized Ohm's law is important. The generalized Ohm's law of MHD is derived from the electron momentum equation, and to a good degree of approximation can be used to justify the statement that the magnetic field is "frozen" to the electron fluid. This is also the case here for (7), which is a modified version of the generalized Ohm's law that includes electron-neutral collisions. We have already noted that in general ν_{ei} and $\nu_{en} << \Omega_e$ for the Earth's ionosphere and magnetosphere, and furthermore, $F_{e\perp} << neE'_\perp$ (again making note of the exceptions to this condition in thin current sheets within the plasma sheet, and regions where reconnection is occurring). Thus (7) can be rewritten as

$$\mathbf{E}'_\perp + \mathbf{U}'_e \times \mathbf{B} = 0 . \qquad (19)$$

In other words, although we formally included electron collisions, they are generally unimportant for the perpendicular component of the modified generalized Ohm's law, and the assumption of frozen-in electrons is usually valid for both the ionosphere and magnetosphere. Electron collisions cannot be ignored for the parallel current density, however.

An important consequence of (19) is, of course, the equivalence between electromagnetic and mechanical loads. Taking the dot product of (19) with the current density, we find that in the frame of the neutrals

$$\mathbf{j}_\perp \bullet \mathbf{E}'_\perp = \mathbf{U}'_{i\perp} \bullet (\mathbf{j}_\perp \times \mathbf{B}) , \qquad (20)$$

since $\mathbf{U}'_{e\perp} = \mathbf{U}'_{i\perp} - \mathbf{j}_\perp / ne$.

The left-hand side of (20) gives the rate at which electromagnetic energy is converted to mechanical energy (positive for an electromagnetic load), while the right-hand side gives the rate at which work is done by the Lorentz force, that is, the rate of work done by the magnetic field (positive for a mechanical load). In the ionosphere the left-hand side of (20) is always > 0 in the frame of the neutrals (as can be seen from (18), again noting that $F_{e\perp} << neE'_\perp$ in the ionosphere), and as a consequence, the ionosphere is a mechanical load on the magnetospheric plasma. In other words, the magnetosphere must "drive" the ionosphere in this frame. Let us be clear here about what is meant by this statement. The magnetospheric plasma must give up mechanical energy to the magnetic field, and the magnetic field in turn does work on the ionosphere to drive the ionospheric convection. This does not mean that parts of the magnetosphere are not themselves loads, but any such loads are connected to other generator regions within the magnetosphere or magnetopause where $\mathbf{j}\bullet\mathbf{E} < 0$.

The statement that the magnetosphere is a driver of the ionosphere is, of course, frame dependent, since a change of frame adds the same constant to both sides of (20), but the equivalence given by (20) is not. Thus provided we can assume that the magnetic field is "frozen" to the electron fluid (a generally good assumption as we have shown here), the ionosphere cannot simultaneously be an electromagnetic load and a driver of magnetospheric convection.

How can this conclusion be reconciled with the examples cited in our introduction, where the ionosphere appeared to drive the magnetosphere? *Siscoe* [1982], in discussing his model, noted that energy transfer between the region 1 and region 2 current systems occurs by enveloping the region 2 current system in the fringing field from an enhanced potential applied to the region 1 system. This is the clue as to how region 2 energization could occur, since the effect of the fringing field is to transfer Poynting flux horizontally from the region 1 system to the region 2 system. Some of the Poynting flux not absorbed through ionospheric Joule dissipation could then travel out along field lines to energize the partial ring current, which closes the region 2 currents within the magnetosphere.

To make this point clear, from (20),

$$\mathbf{j}_\perp \bullet \mathbf{E}'_\perp = \mathbf{U}'_{i\perp} \bullet \left(\frac{(\mathbf{B}\bullet\nabla)\mathbf{B}}{\mu_0} - \nabla\frac{B^2}{2\mu_0}\right), \quad (21)$$

while Poynting's theorem states

$$\mathbf{j}_\perp \bullet \mathbf{E}'_\perp = -\nabla\bullet\mathbf{S}' - \frac{\partial}{\partial t'}\frac{B^2}{2\mu_0}, \quad (22)$$

where $\mathbf{S}'$ is the Poynting flux, and we have ignored the displacement current in (21) and (22).

The first term on the right-hand side of (21) corresponds to field-aligned Poynting flux. This term is positive when downward field-aligned Poynting flux is dissipated within the ionosphere. The second term on the right-hand side of (21) corresponds to the horizontal transport of magnetic energy. Strictly speaking, $\mathbf{U}'_{i\perp}\bullet\nabla B^2/2\mu_0$ is only identical to the divergence of horizontal Poynting flux for time-stationary magnetic fields, although it is often convenient to think of the transport of magnetic energy as being synonymous with Poynting flux. Thus, since $\mathbf{j}_\perp\bullet\mathbf{E}'_\perp > 0$ in the ionosphere, any net upward Poynting flux within a flux tube of necessity requires a net horizontal transport of magnetic energy into the flux tube.

Returning to *Siscoe*'s [1982] model, it is not clear why any Poynting flux should in fact travel out of the ionosphere, and further the assumed ionospheric potential structure sidesteps the question of how the ionosphere is set into motion. Nevertheless, the physical principles involved show how the ionosphere might appear as a generator to the magnetosphere. In Siscoe's model, upward Poynting flux may be present when the region 2 field-aligned currents become enveloped within the flow field of the enhanced region 1 system. No such interconnected flow is present in the work by *Song et al.* [1999, 2000]. For steady state the ionospheric flows in their simulation form a four-cell pattern, and while magnetic energy may be transported around the cells, there is no transport of magnetic energy from one cell to another. In particular, the two-cell pattern of ionospheric flows associated with the high-latitude northward B_z current system does not encompass the lower latitude region 1 sense field-aligned currents, unlike Siscoe's model where high-latitude region 1 driven flows encompass the lower latitude region 2 current system. There is hence no horizontal transport of magnetic energy into the lower latitude current system, and no source of upward Poynting flux to drive the magnetosphere at lower latitudes.

The discussion laid out here also supports the arguments set forth by *Lockwood and Cowley* [1999]. In particular, the ionosphere drives the magnetosphere only if there is upward Poynting flux, and this requires a horizontal gradient in the magnetic energy density, as shown by (21). At the same time the ionospheric incompressibility invoked by *Ridley et al.* [1999] to explain the "prompt" ionospheric response implies only small gradients in the magnetic energy density. Given the highly dissipative nature of the ionosphere, it therefore appears unlikely that there is sufficient horizontal flux transport to both overcome the Joule dissipation and supply the necessary upward Poynting flux. It is more likely that downward Poynting flux from the magnetosphere supplies the energy lost through ionospheric dissipation, supporting the position that the magnetosphere drives the ionosphere. Why the ionospheric response appears to be prompt and widespread may a consequence of induction electric fields shielding the ionosphere from magnetospheric changes until the required current systems have been fully established (N. C. Maynard, personal communication, 2000).

5. Summary and Conclusions

In summary, we have clearly demonstrated that the magnetic field is frozen-in to the electron fluid in both the magnetosphere and the ionosphere as a consequence of the generalized Ohm's law. Furthermore, the momentum equation determines the current density. In the magnetosphere the current density is obtained by balancing the Lorentz force with non-electromagnetic forces, while in the ionosphere the Lorentz force balances the drag force caused by ion-neutral collisions. Because of the frozen-in condition for the electron fluid, there is a direct equivalence between electromagnetic and mechanical loads. This equivalence may appear to break in circumstances where Poynting flux flows out of a load, as if the load was an electromagnetic generator, but by necessity there must be transport of magnetic energy into the load from another direction such that there is a net inward Poynting flux to balance the dissipation within the load. The statement that upward Poynting flux is required for the ionosphere to drive the magnetosphere casts doubt on the assertion of *Ridley et al.* [1999] that the ionosphere drives the magnetosphere in response to changes in the IMF. In addition, *Song et al.* [1999, 2000] do not provide a case where the ionosphere drives the magnetosphere since there is no horizontal transfer of Poynting flux from one current system to the other in their simulations.

Acknowledgments. One of us (R.J.S.) wishes to acknowledge several conversations with P. Song, whose work with global MHD simulations motivated us to reinvestigate the basis of MHD in the collisional regime. This work was supported by NASA grant NAG5-3596 and NSF grant ATM 97-13449 to the University of California. This is IGPP publication number 5493.

Hiroshi Matsumoto thanks the referees for their assistance in evaluating this paper.

References

Barakat, A. R., and R. W. Schunk, Transport equations for multi-component anisotropic space plasmas: A review, *Plasma Phys.*, *24*, 389–418, 1982.

Boyd, T. J. M., and J. J. Sanderson, *Plasma Dynamics*, Nelson, London, 1969.

Deng, W., T. L. Killeen, A. G. Burns, and R. G. Roble, The flywheel effect: Ionospheric currents after a geomagnetic storm, *Geophys. Res. Lett.*, *18*, 1845–1848, 1991.

Green, H. S., Ionic theory of plasmas and magnetohydrodynamics, *Phys. Fluids*, *2*, 341–349, 1959.

Haerendel, G., Field-aligned currents in the Earth's magnetosphere, in *Physics of Flux Ropes, Geophys. Monogr. Ser.*, vol. 58, edited by C. T. Russell, E. R. Priest, and L. C. Lee, pp. 539–553, AGU, Washington, D. C., 1990.

Kelley, M. C., *The Earth's Ionosphere: Plasma Physics and Electrodynamics*, Academic, San Diego, Calif., 1989.

Knight, S., Parallel electric fields, *Planet. Space Sci.*, *21*, 741–750, 1973.

Lockwood, M., and S. W. H. Cowley, Comment on "A statistical study of the ionospheric convection response to changing interplanetary magnetic field conditions using the assimilative mapping of ionospheric electrodynamics technique" by A. J. Ridley et al., *J. Geophys. Res.*, *104*, 4387–4391, 1999.

Luhmann, J. G., Ionospheres, in *Introduction to Space Physics*, edited by M. G. Kivelson and C. T. Russell, pp. 183–202, Cambridge Univ. Press, New York, 1995.

Ridley, A. J., G. Lu, C. R. Clauer, and V. O. Papitashvili, Reply, *J. Geophys. Res.*, *104*, 4393–4396, 1999.

Scudder, J. D., P. A. Puhl-Quinn, F. S. Mozer, K. W. Ogilvie, and C. T. Russell, Generalized Walén tests through Alfvén waves and rotational discontinuities using electron flow velocities, *J. Geophys. Res.*, *104*, 19,817–19,833, 1999.

Siscoe, G. L., Energy coupling between regions 1 and 2 Birkeland current systems, *J. Geophys. Res.*, *87*, 5124–5126, 1982.

Song, P., D. L. DeZeeuw, T. I. Gombosi, C. P. T. Groth, and K. G. Powell, A numerical study of solar wind – magnetosphere interaction for northward interplanetary magnetic field, *J. Geophys. Res.*, *104*, 28,361–28,378, 1999.

Song, P., T. I. Gombosi, D. L. DeZeeuw, K. G. Powell, and C. P. T. Groth, A model of solar wind–magnetosphere–ionosphere coupling for due northward IMF, *Planet. Space Sci.*, *48*, 29–39, 2000.

J. Raeder and R. J. Strangeway, Institute of Geophysics and Planetary Physics, University of California, 405 Hilgard Ave., Los Angeles, CA 90095-1567. (strange@igpp.ucla.edu)

(Received February 1, 2000; revised June 12, 2000; accepted August 7, 2000.)

Paper number 2000JA900116.
0148-0227/00/2000JA900116$09.00