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THE THICKNESS OF THE MAGNETOSHEATH: CONSTRAINTS ON THE POLYTROPIC INDEX

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Abstract. A statistical analysis of 351 independent bow shock crossings and 233 independent magnetopause crossings by the ISEE-1 spacecraft from 1977 to 1980 was performed to determine the average positions and shapes of the bow shock and magnetopause. The standoff distance between the magnetopause and the bow shock depends on the compressibility of the plasma which in the "polytropic" approximation is related to the ratio of specific heats, γ. Standoff distances for the bow shock and magnetopause were found to be 13.7 R_E (± 0.2 R_E) and 10.3 R_E (± 0.3 R_E), respectively. These distances are smaller than those observed during earlier epochs. The observed thickness of the magnetosheath is that expected for the compression of a gas whose polytropic index, γ, is 1.76 ± 0.15. This value, representative of the entire magnetosheath, is consistent with the value of 1.67 deduced from the behavior of the plasma across individual shock transitions. A value of 1.67 is expected for an adiabatic process in a collisional, monatomic gas with three degrees of freedom with lower values for non-adiabatic processes and higher values for anisotropic heating at the shock. The observed value of 1.76 indicates that heat flux does not much affect the position of the shock while the downstream anisotropy may have a small effect.

Introduction

In the Rankine-Hugoniot equations for predicting the jump in magnetic field strength across collisionless shocks, the polytropic index, γ, appears as part of an equation of state for the plasma in order to close the conservation relations. This "arbitrary" parameter is determined by the physical processes present. In an adiabatic process in a collisional gas, this index is the ratio of specific heats and reflects the numbers of degrees of freedom the plasma exhibits. It can be shown from thermodynamics that:

$$\gamma = \frac{f+2}{f} \tag{1}$$

where f is the number of degrees of freedom of the plasma. In a collisionless magnetized plasma the ionized particles gyrate about the ambient magnetic field and the heating may take place only perpendicular to the magnetic field. Thus, it is possible that an adiabatic plasma may exhibit only two degrees of freedom, those perpendicular to the magnetic field, corresponding to γ=2, as opposed to the three degrees of freedom of a monatomic gas, for which γ=5/3. Moreover, if the process is not adiabatic, the polytropic index may be quite different than that expected for an adiabatic process. For example, in an isothermal process, the polytropic index is unity.

Several studies have been performed to determine the proper polytropic index of the solar wind and postshock plasma. In these studies, the polytropic equation of state was used to close off the moment equations in the Rankine-Hugoniot relations. The measured downstream parameters of the plasma were then compared with the values predicted by the Rankine-Hugoniot conservation relations [e.g., *Tidman and Krall*, 1971; *Chao and Goldstein*, 1972] as a function of the polytropic index. These studies used the jumps observed across interplanetary shocks [*Russell et al.*, 1983], across the Venus bow shock [*Tatrallyay et al.*, 1984], and across the Earth's bow shock [*Winterhalter et al.*, 1985]. Each used measured upstream parameters of the solar wind plasma to predict the parameters downstream, then varied the polytropic index to bring this predicted set of values into agreement with those measured by the spacecraft. In the reports by *Russell et al.* [1983] and *Tatrallyay et al.* [1984], respective values for γ of 5/3 and 1.85 were determined, while the large statistical study by *Winterhalter et al.* [1985] found a value of γ=5/3 best fit the observations.

A second procedure for determining the ratio of specific heats is to locate the position of the detached bow shock in front of an impenetrable obstacle. This position depends both on the compressibility of the plasma at the shock and throughout the magnetosheath, which may be different. *Spreiter et al.* [1966] used a gas dynamic model to determine the standoff distance of a detached bow shock while varying the polytropic index and the Mach number of the oncoming flow. The ratio of the downstream to upstream densities can be expressed by [*Spreiter et al.*, 1966]:

$$\frac{\rho}{\rho_{sw}} = \frac{(\gamma+1)M_{MS}^2}{(\gamma-1)M_{MS}^2+2} \tag{2}$$

where γ is the polytropic index of the plasma and M_{MS} is the upstream magnetosonic Mach number of the solar wind. Using results of numerous numerical solutions for supersonic flow past a blunt-nosed obstacle, an empirically derived relation for the standoff distance of a detached bow shock was obtained. This relation is [*Spreiter et al.*, 1966]:

$$\frac{\Delta}{D} = 1.1\frac{\rho_{sw}}{\rho} \tag{3}$$

where Δ is the standoff distance of the bow shock from the nose of the magnetosphere and D is the standoff distance of the magnetopause from the center of the Earth. Combining equations (2) and (3), one can estimate the polytropic index

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γ, given the subsolar distances of the magnetopause and bow shock and the upstream magnetosonic Mach number. This relation for γ becomes:

$$\gamma = \frac{(1.1 + \frac{\Delta}{D}) M_{MS}^2 - 2.2}{(1.1 - \frac{\Delta}{D}) M_{MS}^2} \quad (4)$$

The factor 1.1 in equation (3) is an empirical coefficient based on numerical solutions for supersonic flow past a blunt-nosed obstacle over a wide range of Mach numbers and polytropic indices. To avoid this empiricism *Zhuang and Russell* [1981] examined an analytical solution for a spherically symmetric obstacle. They also assumed that the parameters directly behind the shock were constant along radius vectors. The results of this model were similar to that of *Spreiter et al.* [1966] but, as expected, not identical. The magnetosheath thickness was found using early ISEE 1 and 2 observations and found best to agree with the model if the polytropic index were 2. This conclusion was in contrast to a value close to 1.67 found from analysis of individual shock crossings.

The purpose of this study is to re-examine the paradox raised by the *Zhuang and Russell* [1981] study and to estimate the polytropic index γ utilizing the empirical relation of *Spreiter et al.* [1966]. ISEE-1 bow shock and magnetopause crossings are used to statistically determine the positions of the respective boundaries. ISEE-1 measurements are also used to estimate the magnetosonic Mach number at the times of boundary crossings. From these values the ratio of specific heats can be obtained.

Procedure

ISEE-1 magnetopause and bow shock crossings from the start of the mission in 1977 to the end of 1980 were used to determine the respective positions of these boundaries. Bow shock crossings were determined from magnetic field measurements and magnetopause crossings are taken from *Song et al.* [1988]. Various data reduction and normalization techniques were utilized to increase the statistical significance of this data set.

Due to the variability of the positions of the magnetopause and the bow shock, there are many instances when the spacecraft may encounter these boundaries quite often in a short period of time. If all crossings were used in the analysis, this would bias the data set toward these regions where the spacecraft spends a long period of time close to these boundaries. For this reason, the set of magnetopause and bow shock crossings were separately reduced to a set of median points. For any independent group of multiple crossings, this group is reduced to a single crossing in which the median crossing represents the group. Median points which represents a group of multiple crossings and single individual crossings have the same weight in the analysis. All positions of remaining points, measured in GSE coordinates, are then corrected for the aberration of the solar wind. A 3.8° aberration from the Sun-Earth line is used to correct positions of all magnetopause and bow shock crossings. We then normalize all positions with respect to the solar wind dynamic pressure, ρv^2. It is known that the position of the magnetopause and the bow shock vary as the inverse one-sixth power of the dynamic pressure [*Binsack and Vasyliunas*, 1968], thus we normalize all positions to a reference value of the dynamic pressure using this relation. A reference pressure of 1.80 nPa is used to normalize the respective boundary positions. This value is the median dynamic pressure value for all independent bow shock and magnetopause crossings during this time period.

We further reduce the data set by only using bow shock crossings made by the ISEE-1 satellite for large upstream magnetosonic Mach number. As expected from equations (2) and (3) and observed with Pioneer Venus Orbiter data [*Tatrallyay et al.*, 1983], the position of the bow shock varies with magnetosonic Mach number. At low Mach numbers, the bow shock moves further away from the obstacle to properly divert the flow. When M_{MS}>4.5, the position with respect to Mach number stays relatively constant, so we used only crossings for which the Mach number is greater than this value. The solar wind Mach number at 1 AU is most often above this value, so this restriction does not greatly reduce our number of observations.

As a result of the preceding data reduction and normalization procedures, a set of 351 independent bow shock crossings and 233 independent magnetopause crossings were obtained for the period from 1977 to 1980. The normalized positions in X and R are known, in which X represents the distance from the center of the Earth to the projection of the boundary crossing on the aberrated Sun-Earth line and R is the distance from the center of the Earth to the crossing position. For each boundary the respective normalized crossing positions were divided into 10° bins of solar zenith angle ψ and the medians of X and R were found for each bin. If we assume that the bow shock and magnetopause can be approximated by ellipsoids of revolution, the equation:

$$R = \frac{\kappa}{1 + \varepsilon \cos\psi} \quad (5)$$

where ε is the eccentricity of the symmetric ellipsoid and κ is the terminator distance, can be reduced to $R = -\varepsilon X + \kappa$, since $X = R\cos\psi$, and a simple regression can be performed. The simple regression was performed on the bin median points, which were weighted according to the number of points in each bin.

Once values for the terminator distance and eccentricity are obtained for the respective boundaries, the standoff distance can be calculated for each. The nose of the ellipsoid model (cos ψ=0) is:

$$D = \frac{\kappa}{1 + \varepsilon} \quad (6)$$

where D stands for the standoff distance of the boundary from the center of the Earth. This is the procedure used to determine the parameters of the magnetopause and the bow shock.

Errors reflect the scatter of the normalized points used in the analysis from the fit made by the simple regression. The standard errors for the terminator distance, κ, and the eccentricity, ε, are calculated by finding the square root of the sum of the squares of the differences between each normalized position and the regression fit and dividing it by the square root of the number of independent points minus two ($\sqrt{n-2}$). All errors for parameters that were calculated from the statistically determined terminator distance and eccentricity are found by the general method of propagation of errors [e.g., *Bevington*, 1969]. For example, if a parameter z is a function of x and y which have standard errors σ_x and σ_y the general form for the standard error of z, σ_z, is [e.g., *Bevington*, 1969]:

$$\sigma_z = \sqrt{\sigma_x^2\left(\frac{\partial z}{\partial x}\right)^2 + \sigma_y^2\left(\frac{\partial z}{\partial y}\right)^2} \qquad (7)$$

assuming that the covariance σ_{xy} is equal to zero. All errors in this analysis are determined in this general fashion.

Discussion

Boundary parameters for the magnetopause and the bow shock are shown in Table 1. The standoff distances are 13.7 R_E (± 0.2 R_E) and 10.3 R_E (± 0.3 R_E) for the bow shock and the magnetopause, respectively. Figure 1 shows the shapes and sizes of the bow shock and magnetopause together with the independent crossings used to determine the boundaries. These distances are somewhat smaller than those observed in earlier time periods. We attribute this difference to a greater solar wind dynamic pressure during this time than during earlier epochs [*Petrinec et al.*, 1991]. Fairfield [1971] used data from Imp 1-4 as well as Explorer 33 and 35 to determine standoff distances of 11.0 R_E and 14.6 R_E for the Earth's magnetopause and bow shock, respectively. The time span for the study by *Fairfield* [1971] ranged from 1963 to 1968. *Slavin and Holzer* [1981] included Imp 3 and 4, Heos 1, and Prognoz 1 and 2 data to determine a standoff distance of 13.8 R_E for the terrestrial bow shock. The time span for the study by *Slavin and Holzer* [1981] ranged from 1965 to 1972.

Table 2 displays the average dynamic pressure and magnetosonic Mach number for the period from 1977 to 1980, as well as other calculated parameters for the magnetosheath and bow shock. The normalized thickness of the magnetosheath (Δ/D) found in this analysis is 0.33 (± 0.04). This is in excellent agreement with the results of *Fairfield* [1971], who also obtained a normalized thickness of 0.33 for a somewhat larger standoff distance. From these results, we can then estimate the polytropic index γ using equation (4). Table 2 shows that the results obtained correspond to a ratio of specific heats of 1.76 (± 0.15). This is within one standard deviation of γ=5/3 and two standard deviations away from γ=2. Thus, our value for γ derived

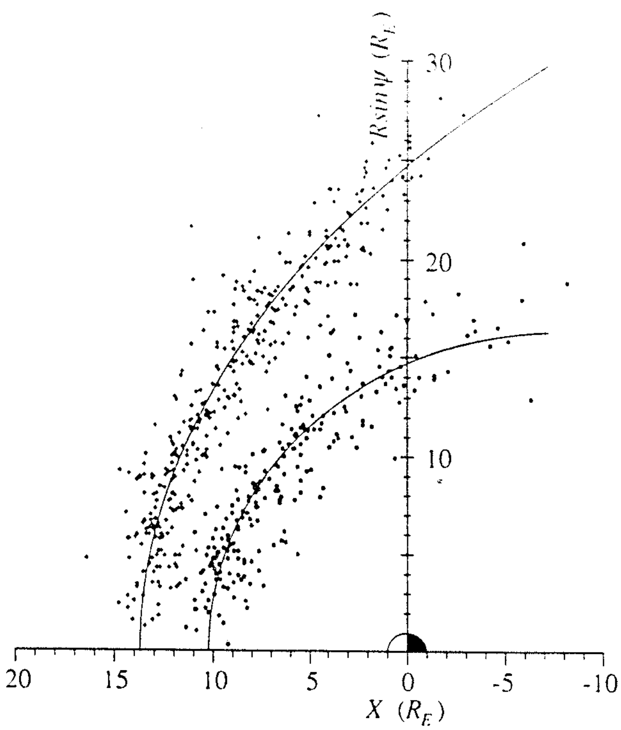

Fig. 1. Ellipsoid model fit to ISEE-1 magnetopause and bow shock crossings for the time period 1977-1980. Circles represent magnetopause crossings and crosses represent bow shock crossings. Units are in Earth radii.

from the global properties of the flow is most consistent with the value of 1.67 derived from studies of the behavior of the plasma across individual shock transitions [e.g., *Russell et al.*, 1983; *Tatrallyay et al.*, 1984; *Winterhalter et al.*, 1985]. A value of 1.67 would be expected for an adiabatic collisional gas with 3 degrees of freedom. A magnetized plasma need not have this ratio. If the shock wave adiabatic and the perpendicular heating across the shock were not isotropized by scattering, we would expect γ to be greater than 5/3, approaching 2 for only perpendicular heating. Thus our results can be interpreted to indicate that the processes acting behind the shock and through the magnetosheath limit the size of temperature anisotropies. If the shock is not adiabatic, the polytropic index can be less than 5/3. In the limit of an isothermal process γ is somewhat greater than 5/3 suggests that heat flux at the shock is on average unimportant in determining the polytropic index. Furthermore, these results show consistency between the polytropic index observed at individual crossings and that deduced from the

Table 1 - Parameters for Earth's bow shock and magnetopause 1977 to 1980

	Bow shock	Magnetopause
Independent crossings	351	233
Ellipsoidal eccentricities	0.81 (±0.02)	0.43 (±0.03)
Terminator distance (R_E)	24.8 (±0.2)	14.7 (±0.3)
Subsolar distance (R_E)	13.7 (±0.2)	10.3(±0.3)

Table 2 - Parameters for the Earth's magnetosheath 1977 to 1980

Average upstream dynamic pressure (nPa)	1.80
Average upstream Mach number	5.4
Subsolar magnetosheath thickness (R_E)	3.4 (±0.4)
Normalized magnetosheath thickness [$\frac{\Delta}{D}$]	0.33 (±0.04)
Polytropic index γ [from equation (4)]	1.76 (±0.15)

magnetosheath thickness in contrast to the study of *Zhuang and Russell* [1981]. Perhaps their conclusion that γ lay closer to 2 was a result of their approximations.

Conclusions

Using 351 independent bow shock crossings and 233 independent magnetopause crossings from the ISEE-1 spacecraft during the time period of 1977-1980, ellipsoidal models of the respective boundaries are obtained. The standoff distances for the bow shock and magnetopause are found to be 13.7 R_E (± 0.2 R_E) and 10.3 R_E (± 0.3 R_E), respectively. These standoff distances are smaller than those determined in an earlier time period (1963-1968) by *Fairfield* [1971]. With these values and the average upstream magnetosonic Mach number for this time period (5.4), the polytropic index γ is calculated using the empirical relation of *Spreiter et al.* [1966]. From this, a ratio of specific heats of 1.76 (± 0.15) is determined, which is less than one standard deviation away from 1.67, the value observed when individual crossings are studied and the value expected for complete isotropization in the magnetosheath. The fact that this value is slightly greater than 1.67 suggests both that heat flux is unimportant in determining the location of the shock and that some weak temperature anisotropy remains downstream but the probable error of the mean of our result is too large for us to conclude this unambiguously.

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