MAGNETIC PERMEABILITY MEASUREMENTS AND A LUNAR CORE

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Abstract. Measurements of the magnetic field induced in the moon while it is in the geomagnetic tail lobes have been interpreted in terms of lunar magnetic permeability due to free iron content; such studies ignored the possibility that a highly conducting lunar core (Fe or FeS) would exclude magnetic fields with an apparent diamagnetic effect. Using lunar chemical and thermal models to determine plausible limits of magnetic permeability, we interpret measurements of the induced moment. The most likely radius of a lunar core is 500 km. Subsatellite and ALSEP measurements of the induced field are in disagreement. Resolving the differences is critical to determining whether a core could or does exist.

The moon's induced magnetic moment, usually attributed to its ferro/paramagnetic properties, has been estimated from orbital and surface magnetometer measurements. Such studies (Parkin et al., 1973, 1974; Dyal et al., 1975; Russell et al., 1974a, 1974b, 1975; hereinafter referred to as PDD73, PDD74, D74a, R74a, R74b, R75) use data obtained in the high latitude tail lobes of the earth's geomagnetic field. In the lobe regions, the magnetospheric field is considerably larger in magnitude and less fluctuating than in other plasma environments, providing favorable conditions for observation of the long time scale lunar electromagnetic response. Interpretations of such data in terms of lunar permeability and free iron content have not considered the possibility of a highly conducting lunar core. A core of Fe-FeS (Brett, 1973) composition would almost completely exclude magnetic field for times typical of major changes in the tail field with an apparent "diamagnetic" effect. As two quantities (core exclusion and bulk permeability outside the core) determine one observable property (the induced magnetic moment), we discuss chemical and thermal limitations on global permeability. Observations of the induced magnetic field then place limits on the radius and conductivity of the lunar core.

We assume that the moon is immersed in a uniform external magnetic field, i.e., the geomagnetic tail lobe field. The magnetic permeability of the external environment is taken to be that of free space; plasma effects will be discussed later. The moon is modeled as a spherical, multi-layered body with the different layers having different values of magnetic permeability.

In this case, the induced field observed external to the moon is a dipole field. If the external magnetizing field is denoted by $\mathbf{H}$, then the total magnetic field observed at the lunar surface at time $t$ due to the external and induced fields is denoted by $\mathbf{B}$, then $G(t) = G_\mu - G_{BE}(t)$ where

$$\mathbf{B} = H_x (1+2G) \mathbf{1} + H_y (1-\lambda) \mathbf{y} + H_z (1-\lambda) \mathbf{z}$$

(1)

In this equation $x$ is upward from the lunar surface, $y$ and $z$ are tangent to the surface. $G$ is a parameter (notation of PDD74) for the total induced dipole with contributions from both lunar magnetic permeability ($G_\mu$) and transverse exclusion of the external magnetic field from the conducting lunar interior ($G_{BE}$). $G_\mu$ can be approximated by summing the contributions from each spherical shell (PDD74, eq. 3) if the relative magnetic permeability, $\mu$, of each layer is close to one. Thus,

$$G_\mu = \sum_{i=1}^{N} \frac{1}{3} (\mu_i - 1) (\lambda_i^3 - \lambda_i^2)$$

(2)

where $\lambda_{B1}$ (or $\lambda_{B2}$) is the ratio of the outer (or inner) radius of a shell to the lunar radius and the sum is computed over all layers. Defining $G_1(t)$ to be the conductive lunar response to a step function increase in the external magnetic field ($0 \leq t < 0$, $H$), the total step function response, $G_0(t) = G_\mu - G_{BE}(t)$. For the case of a two-layer model with a nonconductive mantle and a core of finite conducitivity, $G_0$ and $G_\mu$ are a ratio of radius to the lunar radius of $\lambda$, $G_\mu(0) = 1/2 \lambda^2$ whereas for $\tau = \infty$, the value of $G_\mu(\tau)$ approaches zero. For $\delta \mu$ less than $10^{-4}$ mhos/m, exclusion decreases rapidly with time; thus, magnetic permeability measurements obtained more than a few hours after major changes in the external magnetic field would truly represent lunar magnetic permeability. Alternatively, $G_\mu$ as large as $10^3$ mhos/m would result in virtually complete exclusion of the tail field from the lunar core for the duration of a tail crossing.

As previous results from lunar permeability studies have differed (PDD74, R74b), we employ two different methods of determining the lunar electromagnetic response: a) comparison of Apollo 12 and Explorer 35 data, and b) observation of the dipole field configuration by orbiting subsatellites. Our study confirms that the methods do give significantly different results. We now briefly describe our analysis methods; a complete description will be published elsewhere.

The technique used in this and prior studies of lunar magnetic permeability is to choose magnetically quiet periods (less than 1 y change
during an orbit for the subsatellite technique) in the tail lobes, wait some time (3 hours in our case) for the short period response \( G_b(t) \) for small \( t \) to decay, and then obtain an estimate, \( G_{obs} \), of the induced field. This method greatly reduces the short period contribution to \( G_{obs} \) from \( G_b(t) \). Averaging all measurements of \( G \) over observing periods, \( \Sigma \) (eq. 3), and expressing \( B_X \) as a convolution of the lunar response, \( G_b \), with past variations of \( H_x \), a weighting function, \( W(t) \) is defined such that

\[
G_{obs} = \int_G G(t) \, dt / \int_G dt = \int_0^\infty G_b(t) \, W(t) \, dt \quad (3)
\]

\[
W(t) = \frac{1}{\int H(t) \, dt / \int H \, dt} \int H(t) \, dt / \int H \, dt . \quad (4)
\]

Using the solar magnetospheric X component of the field at two hour intervals, making no corrections for changes in field orientation or plasma sheet diamagnetism, and replacing missing data by previous values, we have calculated the weighting function. For the subsatellite data the value in the interval 0 to 2 hours is 0.011, indicating that the short period response should be suppressed and that there was a slight average tendency for the field magnitude to decrease within 2 hours before observing periods.

Our first determination of \( G \) is by comparison of Apollo 12 and Explorer 35 data. Our method is essentially that of PDD74 except that we use two-hour averages computed from low-pass-filtered ten-minute averages, and reject data that is variable at 20-minute to 2-hour periods. Typical results were \( G_{obs} = 0.009 \pm 0.009 \) based on 37 averages and \( G_{obs} = 0.005 \pm 0.005 \) based on 19 averages. In all cases the estimated value of \( G_{obs} \) was in the range 0.005 to 0.010. Smaller values of \( G_{obs} \) were associated with smaller error estimates. A similar mean-standard deviation relation between the estimates of PDD73 and PDD74 also occurs. This strongly suggests that a variable source of bias is present at times. The estimate and error (0.004 \( \pm \) 0.002) obtained by PDD74 suggests a more successful elimination of periods contaminated by plasma diamagnetism, possibly due to their criteria rejecting data with short period (\(<1 \) min.) or more) fluctuations.

Our second method of estimating \( G_{obs} \) uses Apollo 15 and Apollo 16 subsatellite magnetometer data obtained in low lunar orbit while in the tail lobes. A study of the global lunar magnetic field (R74b, R75) indicated that the induced lunar dipole moment opposed the external magnetic field (in contrast to PDD74, PDDH, and our first method). We discuss the significant differences of our present approach from that of R74b. To estimate \( G_{obs} \), the average value of the magnetic field is subtracted from the data for a given orbit; the residuals are rotated into a coordinate system in which \( B_p \) is radially outward, \( B_t \) is tangent to the orbit, and \( B_n \) is normal to the orbit plane. The residuals are normalized by dividing by the magnitude of the average field component in the orbit plane, defining an azimuthal angle, \( \theta \), along the orbit from the projection of the magnetic field onto the orbit plane, the field components are fit to \( \sin \theta \) and \( \cos \theta \) Fourier components. The dipole is obtained from the \( \cos \theta \) + \( B_p \sin \theta \) estimate. For each orbit, an error estimate is computed from the root mean square average of the following quantities: 1) the discrepancy in \( G_{obs} \) as determined from \( B_p \cos \theta + B_p \sin \theta \), 2) the apparent dipole at right angles to the average field, \( B_p \sin \theta \), and 3) the orbit periodic variation of the component per pendicular to the field, \( B_p \cos \theta \). The component of the external magnetic field normal to the orbit plane causes induced field components that are also normal and constant along the subsatellite orbit.

In the present study, orbits within 3 hours of leaving the plasma sheet or magnetosheath are discarded, as are orbits contaminated by unusually large low energy particle fluxes. The plasma environment is measured by the SISE detector (Hardy et al., 1975). Orbits for which the error estimate of \( G_{obs} \) is greater than 0.025 are also discarded (about 10% of the remaining data); such discarded estimates are strongly aliased by temporal variations of the tail field and probably the values of \( G_{obs} \) are averaged over all orbits; the previous work averaged equally north lobe and south lobe estimates. The value obtained by this procedure is \( G_{obs} = -0.0093 \pm 0.0023 \) based upon all data from the Apollo 15 and 16 subsatellites that met the criteria (27 orbits).

Averaging all values of \( G_{obs} \) assumes (in effect) the time-independent behavior of a perfectly conducting core surrounded by a perfectly insulating mantle. A time-dependent analysis is also possible. \( G_b(t) \) is approximated by the functional form of three different constant values for \( t = 0 \) to 12 hr., \( t = 12 \) to 30 hr., and \( t > 30 \) hr. The three values are then solved for by a least squares fit of observations of \( G_{obs} \) to predictions of \( G_{obs} \) from eq. 3 after obtaining \( W(t) \) from eq. 4. The estimates are shown in Fig. 1; the estimate assuming constant \( G_b \) (average value) is similarly plotted. Also shown is the calculated response of a 500 km radius lunar core surrounded by an insulating mantle for core conductivities of 0.1, 1.0, and 10\(^2\) mhos/m.

The previously reported disagreement between estimates of \( G_{obs} \) obtained using surface data (PDD74, PDDH, and subsatellite measurements (R74a, R74b, R75) is confirmed by our study. Various sources of bias that might affect the subsatellite observations are: 1) a permanent lunar dipole field, 2) plasma diamagnetism, 3) the conductive response of the lunar mantle, and 4) instrumental effects. The permanent lunar dipole possibility has been investigated in detail (R7Ha, R7Hb, R75); no evidence for a permanent dipole field was found. Plasma inertial effects will tend to compress a lunar dipole field; based upon a dimensional scaling we estimate this effect to be on the order of \( 35\% \) to \( 20\% \) for plasma densities of 0.005 cm\(^{-3}\) to 0.03 cm\(^{-3}\) for plasma velocities of 150 km/sec. An estimate of plasma diamagnetism due to thermal effects is obtained by magnetic pressure balance between a plasma and vacuum thus, \( B_p \rho_0 = \sqrt{1 + B_p^2/\rho_0^2} \) where \( B_p \) is the plasma pressure, \( \rho_0 \) in the vacuum field, and \( \mu_0 \) is the "magnetic permeability" of the plasma. For plasma densities of 0.05 cm\(^{-3}\)
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Figure 1. Observations and model values of $G_R(\tau)$ (assumed if SIDEC detects no plasma) to 0.3 cm$^{-3}$ (a typical tail value) and a temperature of 15 electron volts, we obtain $\delta G = 1/3 (p_{P}\rho_0 - 1) = -0.0005$ to -0.003, where $\delta G$ is the apparent increase in lunar magnetic permeability. Simple plasma diamagnetic increases (rather than explains) the discrepancy between subsatellite and surface observations. Despite the narrow view angles of the SIDEC detectors, both the Apollo-14 and Apollo-15 detectors usually agree as to whether plasma is present; also, a high correlation exists between detection of plasma and the East-West component of the interplanetary magnetic field. We conclude that SIDEC is an effective monitor of the plasma environment except during periods of solar directed plasma sheet flow. We have used SIDEC data to determine periods when plasma is present; future studies should also use FIPS electron data. Plasma diamagnetic effects have also been analyzed by separately averaging only those 12 orbits during which no plasma was observed. For the data reported here we obtain a value of $G_{obs} = -0.0062 \pm 0.0045$. The apparent diamagnetic effect is more than three times larger than estimated above, but is not statistically significant. Other observational evidence (Goldstein and Russell, 1975) shows no plasma effects.

The conductive response of the lunar mantle might also affect our results; for the lunar interior model to be discussed this effect is unimportant and TEC node due to mantle conductivity contribute only random error.

Finally, we note that temperature-dependent effects in the subsatellite data that have been calibrated on the ground are very small, and have been corrected in data processing. Neither will a lunar ionosphere affect magnetospheric measurements (Goldstein and Russell, 1975). There may be some biases in our measurements and we estimate a maximum plausible errors in the value of $G_{obs}$ of 0.003 due to mantle conductivity and plasma diamagnetism.

Our estimate of induced moment using comparisons of Apollo-17 and Explorer 35 data suggests substantial influences of plasma diamagnetism and/or spatial gradients between the magnetometers. However, the Apollo-15 and -16 comparison (EWD) used FIPS data to eliminate periods when plasma was observed. Several other sources of error which might affect surface magnetometer observations are: local permeability anomalies, compression of local remnant fields by ambient plasma and magnetometer drifts. However, studies using different data sets (Explorer 35 and Apollo 12 LEM data, FWD74, Apollo 15 and 16 LEM data, BD74) are in good agreement. We conclude that the cause of the discrepancy between the surface and subsatellite measurements is unknown and that no definitive reason exists for choosing between the measurements.

Contributions to the very long period lunar response would be expected from both lunar magnetic permeability and a highly conducting core. FWD74 show that only ferromagnetism due to free iron below the Curie temperature is important. The lunar interior temperature profile that determines the Curie temperature depth can be estimated from the electrical conductivity of the lunar interior (Dyke et al., 1974b, 1975). A possible depth of the Curie point of about 250 km is suggested; however, compositional changes at depth may affect these estimates. Thermal history models suggest a depth of 350 km (refs. in BD74).

If the amount of free iron in the upper mantle below 60 km is assumed to be typical of whole moon values before differentiation, then plausible free iron concentrations are from 4 to 6% by weight (Ganapathy and Anders, 1974); other geochemical estimates are typically lower. We adopt a differentiated lunar model with no free iron at depth less than 60 km, and with a constant percentage by weight, q, from 60 km to the Curie depth. The magnetic permeability of the upper mantle can then be approximated by $\mu = 1 + 0.0153 q$ (FWD74). Using eq. 2, $q = 6.0$, and a Curie depth of 250 km, the maximum likely value of $G_{T}$ is 0.008. For the rest of this discussion, we assume that $G_{T}$ lies in the range 0.000 to 0.008. A Curie depth of 350 km and a free iron content as large as 6% would increase $G_{T}$ to 0.012 and allow a somewhat larger core.

An estimate of lunar core size is now possible. For a model with an insulating mantle and a perfectly conducting core, values of G for fixed values of $G_{T}$ are shown in Fig. 2 as a function of core radius; also shown are measurements from this and other studies. Subsatellite estimates are shown as (a), with the solid brackets indicating statistical error, and the dashed brackets including possible estimated biases. These results require a core with an upper (lower) limit for radius of 350 (350) km for $G_{T} = 0.008$ (0.000). (We assume that any core is towards more negative values of $G_{obs}$.) A possible 350 to 170 km radius core has been estimated from seismic data (Nakamura et al., 1974), but a core as large as 500 km could be compatible with the seismic data. (FWD74 (b), BD74 (a), and our first method (c1, c2) allow a maximum core radius of only about 420 km, but do not require a core.) The minimum core size as determined by subsatellite data will not be substantially reduced from the previous simple estimate by the effects of finite conductivity in the mantle. To study the effects of mantle conductivity, we adopt a three-layer lunar model with constant conductivity within each layer. The core conductivity is taken as $10^{5}$ mho/m for an FeS or Fe composition (Johnston and Strens, 1973) (Other less likely high con-
ductivity (≈5 nho/m) materials are a large fraction melt or semiconducting mineral (e.g. albite)). Above the core we assume a partial melt zone with conductivity of 0.1 nho/m (a probable overestimate to exaggerate the contribution of mantle conductivity); melts of from 3% to 5% might cause such conductivities (Waff, 1974). The outer shell consists of non-molten or small-fraction melt material with an assumed conductivity of zero. A fit of individual measurements of \( G_{\text{obs}} \) to the response of the different models was computed and the effects of mantle conductivity were found to be negligible at the long time periods. Applying the estimated maximum plasma correction, we obtain a minimum core radius of 320 km. The suggested small size (0.6 to 2.5% by volume) of an FeS core is not in conflict with a moon depleted in volatiles. Seismic data also suggest that the moon might have a small differentiated core. The subsatellite measurements are in disagreement with other magnetometer results (PD075, DPW) that allow, but do not require a lunar core. Magnetic data establish a maximum core size of about 580 km but other results such as the existence of a core are controversial and depend upon a future resolution of the observational discrepancies.

The present study has provided evidence suggesting a highly conducting lunar core. Further studies are required, using overdetermined measurements (more magnetometers than necessary) and a variety of plasma data. However, present data are both too few and too noisy. The Lunar Polar Orbiter mission provides an opportunity to obtain data from two orbital magnetometers simultaneously, thus avoiding problems of plasma interactions with local remnant fields that might influence surface observations. A plasma detector (such as proposed electron mirroring experiment) would also be necessary to eliminate periods of plasma diurnality. Such instrumentation would resolve the current discrepancy in estimates of induced lunar magnetic moment, and conclusively test present lunar core results.

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References


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