COHERENCE LENGTHS OF UPSTREAM ULF WAVES: DUAL ISEE OBSERVATIONS

G. Le, C. T. Russell, and D. S. Orlowski
Institute of Geophysics and Planetary Physics, University of California, Los Angeles

Abstract. Waves are generated in front of planetary bow shocks by a variety of plasma instabilities and they occur over a wide range of frequencies and wavelengths. In the strongly Doppler shifting environment of solar wind, multiple spacecraft observations are needed to determine unambiguously the wave properties such as frequency, phase velocity, and wavelength. The separations of the spacecraft required for these studies are restricted because the waves have finite coherence lengths. The multiple spacecraft must be located within a coherence length of each other in order that they are studying the same wave. In this study, we have used high time resolution simultaneous magnetic field data from the dual ISEE spacecraft to study the coherence lengths of upstream ULF waves. We examine the cross-correlation between ISEE 1 and 2 observations for different spacecraft separations and determine the coherence lengths for upstream 30-sec waves, 3-second waves and one-Hz waves. We find that the observed coherence lengths are consistent with those estimated from the bandwidth of the spectral peak, and that these lengths vary markedly from less than 100 km to over 1 R\(_{\text{E}}\). In order to study all these wave phenomena, a multiple spacecraft mission such as the upcoming ESA Cluster mission would need to be capable of assuming a wide variety of possible separations.

1. Introduction

Nearly 10 years of ISEE observations have provided us not only with much information about the Earth’s magnetosphere but also the data needed to plan the next generation of solar terrestrial missions. For example, the upcoming ESA Cluster mission is intended to provide multipoint measurements of plasma, fields and waves in the Earth’s magnetosphere and enables us to separate the spatial and temporal variations. It has many conflicting demands on the operations and has limited fuel for rearranging the required spacecraft separations. In order to use the 4 Cluster spacecraft to study a phenomenon, all the spacecraft must each encounter the same feature. In other words the spacecraft separation should be smaller than the coherence length of the phenomenon to be studied. Existing data sets from the dual ISEE spacecraft can help us to define the coherence lengths of many phenomena, and thus, to develop a separation strategy that keeps the spacecraft closer than the coherence length of the phenomenon to be studied. In this paper, we use magnetic field data from the dual ISEE spacecraft to define coherence lengths of various ULF wave phenomena in the Earth’s foreshock region.

There are several different types of upstream waves in the Earth’s foreshock region due to a variety of plasma instabilities. These waves occur over a wide frequency range. In the ultra-low frequency range, upstream waves have been observed in both the electron and ion foreshock regions. In the electron foreshock region, an electromagnetic mode at ULF frequencies has been identified: the so-called one-Hz waves [Fairfield, 1974; Hoppe et al., 1982], also called upstream whistlers [Orlowski et al., 1993]. The source of one-Hz waves is still controversial at present. Fairfield [1974] first ascribed them to whistler mode waves propagating upstream against the solar wind flow with a phase speed generally less than that of the solar wind, but with a group velocity greater than that of the solar wind so that the wave energy could propagate upstream. Then the upstream electrons [Sentman et al., 1983] were proposed to be the source of the one-Hz waves. Recent study by Orlowski et al. [1993] supported that the waves are generated at the bow shock. In the ion foreshock region, ULF waves have been observed at frequencies below the proton gyrofrequency and have been classified into several classes [Greenstadt et al., 1968; Russell et al., 1971; Hoppe et al., 1981; Le et al., 1992]. Near the leading edge of the ion foreshock, ULF waves are nearly sinusoidal with periods near 30 seconds. They typically have amplitudes of a few nT, are primarily transverse, and exhibit predominantly left-handed polarization in the spacecraft frame [Hoppe et al., 1981, 1982; Hoppe and Russell, 1983]. However, ULF waves with periods near 30 seconds are most frequently observed in large-amplitude, highly compressional form deep in the ion foreshock. They steepen into small shocklike waveform and have been called shocklets [Hoppe et al., 1981, 1982]. Discrete wave packets [Russell et al., 1971], which have higher frequency, are often associated with the steepening edges of the shocklets. The 30-sec waves are commonly believed to be generated by upstream ions. The wave instabilities associated with upstream ions have been extensively studied [Barnes, 1970; Sentman et al., 1981; Gary, 1981; Winske and Leroy, 1984; Omi and Winske, 1988]. Another type of foreshock waves has frequency near three seconds and is always right-handed and nearly circularly polarized in the spacecraft frame [Le et al., 1992]. Their generation mechanism is still unknown.

To unambiguously determine wave properties, such as frequency, wavelength, phase velocity, group velocity and direction of propagation, in the strongly Doppler shifting environment of solar wind, one needs simultaneous observations from multiple spacecraft. However, the separations of the spacecraft suitable for these studies are restricted because the waves have finite coherence lengths, which vary with different wave types. In this study, we use magnetic field observations from the dual ISEE spacecraft to determine the coherence lengths of upstream ULF waves, including large-amplitude 30-sec waves, nearly monochromatic 3-sec waves and one-Hz whistler waves. The cross-correlation coefficient between the ISEE 1 and 2 wave observations is a function of lag time [Hoppe et al., 1981] and its maximum gives a measure of the coherence between the wave packets observed at the two spacecraft. We examine how this maximum cross-correlation coefficient between simultaneous
ISEE 1 and 2 observations vary with different spacecraft separation distance and from the scale length of the variation we determine the coherence length of the waves.

The coherence length of a wave can also be estimated roughly from the bandwidth of the spectral peak. The coherence length of a wave will be infinite only if the wave is purely monochromatic, i.e., the wave has a precisely defined frequency and wave number. In general, a wave is a linear superposition of Fourier components over a band of frequencies. If the medium is dispersive, i.e., \( \omega = \omega(k) \), each frequency component will propagate with a different phase velocity. As a result, the wave will form a packet having a finite spatial extent. To provide a physical picture of the coherence length, we consider an over-simplified case, an one-dimensional wave expressed as:

\[
U(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A(k) e^{-i\omega t - ikx} dk
\]

where \( A(k)dk \) is the amplitude of wave in the range \( k \) to \( k + dk \). \( A(k) \) is determined by the wave form at the time when the wave is generated:

\[
A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} U(x,0) e^{-ikx} dx
\]

If the initial wave form \( U(x,0) \) is a finite wave train with coherence length of the order \( \delta x \), \( A(k) \) will be a peak centered at \( k_0 \) with a bandwidth of the order \( \delta k \). Assume \( U(x,0) = A_0 e^{-x^2/\delta x^2} \), i.e., a Gaussian modulated wave as shown in Figure 1, then, we have: \( A(k) = (\delta x A_0/\sqrt{2}) e^{-k^2(k-k_0)^2/\delta k^2/4} \). Thus, \( A(k) \) is centered at \( k_0 \) with a bandwidth of \( \delta k = 2/\delta x \). From this we know that the relation between the coherence length and the bandwidth of spectrum is \( \delta k \delta x \sim 1 \). Using the dispersion relation \( \omega = \omega(k) \), we have: \( \delta \omega = (d\omega/dk)|_{k=k_0} \delta k = v_g \delta k \). Therefore, the coherence length \( \delta x \) can be estimated from the group velocity and the bandwidth of the waves: \( \delta x \sim v_g/\delta \omega = v_g/(2\pi f) \).

2. Observed Coherence Lengths

In this study, we use simultaneous observations from the dual ISEE spacecraft at different separation distances in the Earth’s foreshock region to determine the coherence length of upstream ULF waves, including large-amplitude 30-sec waves, nearly monochromatic 3-sec waves and one-Hz whistler waves.

2.1. Upstream 30-second Waves

In the Earth’s foreshock region, the large-amplitude waves with periods near 30 seconds are the most common wave phenomenon. We have studied previously the coherence length of these waves as a characteristic quantity in determining the extent of the bow shock’s modulation as they are convected downstream by the solar wind [Le and Russell, 1990]. The coherence length has been determined by examining the variation of cross-correlation coefficients between the ISEE 1 and 2 observations as a function of separation distances. In that work, we examined nearly a thousand 4-minute segments of large-amplitude 30-sec waves in the years of 1978, 1979 and 1980 with separation distances varying from a few hundred km to over one Earth radius (\( R_E \)). We found that the coherence of these waves behaves differently along and transverse to the solar wind flow, mainly due to the solar wind convection effect. The cross-correlation coefficients depend mainly on the spacecraft separation transverse to the solar wind flow. Figure 2 is adapted from Le and Russell [1990] and shows the cross-correlation coefficients as a function of the separation transverse to the solar wind flow from this study. The dots in this figure are the average correlation coefficients within each 0.1 \( R_E \) bin range and the error bars represent the standard derivations. It shows that the cross-correlation coefficient drops to \( \sim 0.5 \) when the perpendicular separation is 1 \( R_E \). The exponential fit (the solid line) gives a scale length of 1.5 \( R_E \). Thus the coherence length of 30-sec waves is \( \sim 1 R_E \) transverse to the solar wind flow. This distance is of the order of the wavelength of 30-sec waves [Hoppe and Russell, 1983].

The coherence length of 30-sec waves can also be estimated from the bandwidth of the spectral peak. For a 30-sec wave in it well developed form, i.e., steepened shocklets, the bandwidth of power spectral peak is typically \( \sim 0.01 \) Hz in the spacecraft frame [Le and Russell, 1990]. The wave group velocity in the spacecraft frame is on the order of the solar wind velocity (\( \sim 400 \) km/s). This gives a coherence length of \( \sim 6000 \) km, consistent with the result of the cross-correlation analysis.

2.2 Upstream 3-second Waves

Another type of upstream ULF waves occurs at periods near 3 seconds. They propagate in the general direction of the magnetic field and are very circularly polarized. These waves are nearly monochromatic with very narrow spectral

![Fig. 1. A Gaussian modulated wave packet and its spectrum.](image)

![Fig. 2. Correlation coefficients of upstream 30-sec waves simultaneously observed by ISEE 1 and 2 as a function of spacecraft separation perpendicular to the solar wind flow.](image)
peak, which means they will have relatively large coherence length. Figure 3 shows an example of 3-sec waves and the power spectrum. The bandwidth of the spectral peak is typically ~ 0.01 Hz. Their bandwidth normalized by the peak frequency is very small: $\delta f/f_p < 10\%$. Since the group velocity of 3-sec waves in the spacecraft frame is in the same order of the solar wind speed, we get the coherence length is in the order of 1 $R_E$.

Figure 4 illustrates results from the cross-correlation analysis of simultaneous ISEE 1 and 2 observations of 3-sec waves. It contains all the 3-sec wave events (more than 200) in the years 1977, 1978, 1979 and 1980. The upper and lower panels shows cross-correlation coefficients as a function of spacecraft separation distances along and transverse to the solar wind flow, respectively. The dots in Figure 4 are the average correlation coefficients in each 1/15 $R_E$ bins and the error bars represent the standard derivations of the data points. From Figure 4 we can see that the correlation coefficient is nearly constant when the separation along the solar wind flow is up to 0.5 $R_E$. However, the correlation coefficient decreases rather fast with the separation transverse to the solar wind flow. The scale length given by the exponential fit is 0.9 $R_E$ in the direction transverse to the solar wind flow. Thus, the coherence length is consistent with what we estimated from the bandwidth of the spectral peak.

The coherence length of 3-sec waves is slightly smaller in km than that of 30-sec waves, but different markedly when normalized by the wavelength since these two types of waves occur in very different frequency and wavelength range. The wavelength can be determined from simultaneous observations from ISEE 1 and 2. It is frame independent and can be expressed as $\lambda = \lambda_{ph}/f = \lambda_{ph}(s/c)/f(s/c) = (s \cdot k/\delta t)/f(s/c)$, where $s$ is the separation vector between ISEE 1 and 2, $k$ is the unit wave normal vector and $\delta t$ is the time delay between the two signals. In contrast to the large-amplitude 30-sec waves, the directions of propagation $k$ (with 180° ambiguity) of 3-sec waves can be very well defined from the minimum variance analysis since a plane wave is a good approximation to 3-sec waves [Le and Russell, 1990; Le et al., 1992]. We have used simultaneous observations from ISEE 1 and 2 with time resolution of 0.25 s to determine the wavelength of 3-sec waves and found that the wavelength of 3-sec waves ranges from ~ 500 km to ~ 4000 km and the average and median value is ~ 1600 km. The coherence length of 3-sec waves is of the order of 4 wavelengths.

Fig. 4. Correlation coefficients of upstream 3-sec waves simultaneously observed by ISEE 1 and 2 as a function of spacecraft separation along the solar wind flow (upper panel) and transverse to the flow (lower panel).

Fig. 5. Correlation coefficients of upstream one-Hz waves simultaneously observed by ISEE 1 and 2 as a function of spacecraft separation transverse to the solar wind flow.

Fig. 3. An example of 3-sec waves. The top panel shows the time series of the magnetic field. The bottom panel is the power spectrum of the indicated wave packet.
2.3. One-Hz Whistler Waves

One-Hz waves have small amplitudes (~ 1 nT) and are often observed upstream from quasi-perpendicular shock and near the leading edges of the foreshock [Fairfield, 1968; Hoppe et al., 1981]. They have been identified as oblique whistlers. Comparing with 30-sec and 3-sec upstream waves, they have much higher frequency (~ 10–100 f_i in the plasma rest frame) and smaller wavelength (~ 100 km). From this study, we find that their coherence length is also very small, in the same order of their wavelength. This is illustrated in Figure 5. It contains only a few data points because the number of one-Hz wave events with small separation of ISEE 1 and 2 is very limited. In Figure 5, the correlation coefficient between ISEE 1 and 2 one-Hz waves decreases much faster with increasing spacecraft separation. It drops to 0.5 when the separation perpendicular to the solar wind flow is ~ 100 km. The scale length given by the exponential fit is 0.02 R_E in the direction transverse to the solar wind flow. Similar to the cases of 30-sec and 3-sec waves, the coherence length of ~ 100 km is also consistent with what we expect from the bandwidth of the spectral peak (not shown).

3. Conclusion

We have used the high time resolution simultaneous magnetic field data from dual ISEE spacecraft to study the coherence lengths of upstream ULF waves. We examine the correlation coefficients between ISEE 1 and 2 data for different separations of ISEE 1 and 2 in the Earth’s ion foreshock region. We find that the coherence length varies with different types of ULF waves. The coherence length is related to the bandwidth of the power spectra and the wavelength as expected from the theory. For the large-amplitude ULF waves with periods near 30 seconds and wavelength close to one Earth radius (R_E), the coherence length is of the order of one R_E transverse to the solar wind flow. For nearly monochromatic ULF waves with periods near 3 seconds, we find that the coherence length to be equal to several wavelengths, again equaling about 1 R_E. However for the much shorter wavelength one-Hz waves, the coherence length is ~ 100 km. While all these coherence lengths are consistent with that estimated from the bandwidth, we see that the coherence length in kilometers varies markedly for different wave phenomena.

One of the major issues for operating the Cluster mission is deciding on the separation strategy. One conclusion from this study is that a variety of different separations are needed to study the various wave phenomena in the upstream region. In order to determine the properties of a wave phenomenon one needs to see the same wave with high coherence at all 4 spacecraft. Upstream 30-second waves and 3-second waves can be studied with moderately large separations (~1000 km), but studies of upstream one-Hz waves require the four spacecraft to be spaced rather closely, at ~ 100 km.

Acknowledgments. This work was supported by the National Aeronautics and Space Administration under research grant NAGW-2067.

References


(Received March 26, 1993; revised June 11, 1993; accepted June 23, 1993.)