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EXPERIMENTAL STUDIES OF THE PROPERTIES OF "SIMULATED" UPSTREAM TURBULENCE USING A STATISTICAL MULTIPOINT METHOD

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ABSTRACT

In this report we present a different approach to the multipoint measurement of magnetic fields and plasma. This is called the multi-spacecraft ensemble technique, MET, essentially free of process restrictions, such as linearity and stationarity. We comprehensively discuss the other conditions and limitations intrinsic to this statistical method. We also show the results of the application of the ensemble method to the synthetic data obtained from a hybrid simulation in the region upstream of a quasi-parallel shock. The important implications of the above approach for the CLUSTER mission are discussed.

INTRODUCTION

Recently, there have been several techniques developed specifically for multispacecraft data analysis. See */1-6/* for details. All these techniques have "built in" a common fundamental assumption of stationarity of the plasma process which can be approximated by one planar wave or linear superpositions of many independent planar waves. In this report we present the foundations for and application of a different technique for determination of the properties of fluctuating particles and fields, the so called multi-spacecraft ensemble technique (MET). In contrast to the other techniques, this statistical method requires neither stationarity of a process nor any specific nature of the fluctuating fields. The results of the MET can in principle be interpreted in terms of any relevant theory as long as the underlying assumptions can be experimentally verified. For a stationary and homogenous plasma process, the MET produces results similar to the results obtained directly from linear Vlasov theory and consistent with the results obtained using other single or multispacecraft methods. Moreover, the MET will appropriately handle cases of large amplitude, incoherent, non-planar, fast growing fluctuations that violate basic linearity assumptions. For such a class of non-linear processes the MET will still be able provide methodically correct results. However, those results will yield average values of field/plasma correlations with a finite statistical weight and hence they have to be interpreted differently than the results obtained using any classical method. In section 1 we present the foundations of the MET, invoking the basic concepts of statistical mechanics and random signal processing theory. In section 2 we test the applicability of the MET to the CLUSTER mission by analyzing the synthetic data obtained from 4 virtual spacecraft flying through the turbulence upstream of the quasi-parallel shock simulated by a 1D numerical (hybrid) method .

FOUNDATION OF MULTI-SPACECRAFT ENSEMBLE TECHNIQUE

We consider here a complete ensemble of N systems (e.g. N spacecraft and their vicinities) that are subject to a single classical and finite process $\{\mathbf{q}_k ; k< \infty\}$. The complete ensemble signifies the ensemble containing the full spectrum of states generated by the process. At any given time t_o we can define the 1st order fluctuation of a random quantity $\mathbf{q}_k$ determined by the classical state k (defined by the measured momentum of the plasma and the location of measurement site) as follows:

$$\delta\mathbf{q}_k = \mathbf{q}_k - \langle\mathbf{q}\rangle \qquad 1)$$

where: $EX(\mathbf{q}) \approx \langle\mathbf{q}\rangle \approx \sum_k \mathbf{q}_k n_k$; $EX(\mathbf{q})$ is the mathematical expectancy of $\mathbf{q}$, $\langle\rangle$ means averaging over an ensemble of N systems and n_k is the fraction of systems in the kth classical state. Note that the ensemble average <q> in the form presented above is only an estimate of EX(**q**). Since n_k in a collisionless plasma overall obeys non-Gaussian (non-equilibrium) statistics, the partition function necessary to theoretically calculate probability $P_k \approx n_k$ is unknown . Also it is very difficult to experimentally determine in a general manner the ensemble average <q> by using n_k because we have to consider an ensemble of a small number of systems (spacecraft) such as will be replicated by the CLUSTER. We note that an adequate estimate of <q> is critical for successful application of the MET. There can be two ways to proceed. The first approach, is to derive (or assume) an explicit form of the probability distribution of the states. The second approach is to obtain (or select) an approximate ensemble average by introducing certain concepts, assumptions and limits. In order to obtain a simplified approximate solution for the determination of <q> we use the concept of a representative ensemble.

The representative ensemble, REN{$\mathbf{q}_k$}, in the sense discussed here is by definition an ensemble of systems which is able to reproduce (with sufficient accuracy) the mean values of n-order fluctuations, $\delta\mathbf{q}_k{}^n$, determined by the complete ensemble of systems for a given process. Note that if n=0 we drop n and δ in $\delta\mathbf{q}_k{}^n$. For example the zero-order REN{$\mathbf{q}_k$} approximates well only zero-order mean values and the 1st-order REN{$\mathbf{q}_k$} approximates well only zero-order and 1st order mean values of the fluctuations, etc. The necessary (but not sufficient) condition for representativeness of the ensemble is that the systems belonging to REN{$\mathbf{q}_k$} are in high probability states c.g $0 << P_k < 1$. This condition is consistent with a fundamental assumption of experimental physics stating that the observable physical quantities are those associated states with significant probabilities. If each of the systems belonging to REN{$\mathbf{q}_k$} can be considered to occupy equal and exclusive volumes of phase space, the probability P_k of finding a specific system in any single classical state does not depend on the particular process but is roughly proportional to 1/N. Formation of REN{$\mathbf{q}_k$} significantly simplifies the problem of estimating EX($\delta\mathbf{q}_k{}^n$) and leads to a generally valid expression for the ensemble averaged quantity <$\delta\mathbf{q}^n$> identical to that of the arithmetic mean. However still, full determination of the representativeness of a given ensemble i.e. the minimum number of systems N (spacecraft) , phase space separation between systems (spacecraft), S_{ij},or the minimum volume (spacecraft and vicinity) of each independent system in phase space comprises a complicated problem. This complexity additionally arises because the determination of the independence of systems in the ensemble depends also on the data aquisition methods and measurement techniques used. The temporal, spectral and directional resolutions of measurements dictate whether or not the approximation of the representativeness of the ensemble can be made for a given effective separation (in phase space) of the systems (spacecraft).

Let us consider a single process {$\mathbf{q}_k(f_k)$}with total energy ε . The energy ε is distributed as f_k for the kth system of the ensemble over spatial and temporal scales λ_m and τ_n respectively, such that $\varepsilon = \Sigma_{kmn}\, \varepsilon_k(\lambda_m, \tau_n)$; If Λ [m^{-1}] and Γ [s^{-1}] are spatial and temporal resolutions and σ [$j^{-1}s^{-1}m^{-1}$] is the generalized sensitivity of data aquisition.

We require that the "classical " representative ensemble of systems REN{$\mathbf{q}_k$} satisfy the following:

$$\sigma \,.\, \varepsilon(\lambda_m\;,\;\tau_n) > \Lambda\,\Gamma \qquad 2)$$

$$L_c\,\Lambda > \lambda_m\,\Lambda > 1 \text{ and } \tau_n\,\Gamma > 1 \qquad 3)$$

where: L_c is the spatial bound of the process; There can be several interpretations of the above conditions. On one hand these conditions may be seen as expressions of the minimum process resolution and sensitivity requirements. On the other hand conditions 2 and 3 can be understood as those determining an access by systems in the ensemble to specific classical states. In the "classical " representative ensemble for a given process all high probability states are assumed to be accessible by systems, hence allowing the <> averaging to be replaced (approximated) by the arithmetic mean. Finally the above conditions 2,3 may also be interpreted as a spatio-temporal window on the processes by the given ensemble of systems (spacecraft). If for a given process these conditions are not satisfied, the ensemble cannot be considered as representative. In such a case spurious, and aliasing effects will occur. Therefore, some prior knowledge about the investigated plasma process is crucial for the proper interpretation of the result produced by the MET.

The physical quantity **q** used in the MET can only be a directly measurable quantity such the fields themselves whose fluctuations are determined from the ensemble of spacecraft at the time "instant" t_o (that persists Γ^{-1}).

The quantity $\mathbf{q}_k$ which corresponds to one specific classical state of a system with energy ε_k, may have different values at different measurement times. However, $\mathbf{q}_k$ generally does not have a known (or deterministic) dependence on time. Instead $\mathbf{q}_k$ and $\delta\mathbf{q}_k$ form random sequences (generated by the process {$\mathbf{q}_k$ k<∞}) with time as a index [tn]. Obviously the propagation angle or the polarization (in a strict sense) have to be derived from a time sequence of "events" and therefore, in general, cannot be evaluated by the MET except for the case of a fully deterministic process e.g. in case when all plasma particles move along topologically identical trajectories.

The quantities that can be succesfully evaluated by the MET are the ensemble averages of fluctuating fields, <δ**B**>, <δ**E**>, <δ**F**> where δ**F** = δn.δ**v** and any other <> quantity formed by the combination of these fields such as the Poynting flux <δ**B** x δ**E**>. In this paper we will discuss only selected correlations based on δ**B**, δ**A**, δn, δ**v** where: **∇** x δ**A** = δ**B** such as magnetic helicity, cross-helicity and compressibility. We want to note that quantities evaluated numerically from δ**B** field gradients between systems (spacecraft) may introduce spurious contributions to the derived quantity (δ**A**) when significant power is carried by the scales comparable or shorter than the separation distance S_{ij}. Therefore, for example, the dimensionless helicity derived by the MET may depend on the configuration of systems (spacecraft) and be negatively affected despite satisfying all other methodical conditions.

There is yet another factor that influences our ensemble average approximation e.g. process uncertainity. It is of great importance that most of the systems were subjected to the "same" process. The uncertainity is due to the fact that besides any intrinsic plasma process that may operate, the very existence of the spacecraft perturbs the plasma and sets up its own characteristic process as a result of spacecraft-plasma interactions.

CORRELATION ANALYSIS

In order to investigate the spatial and temporal scales of the classical fluctuations in the plasma we utilize well known cross-correlation functions of the random quantities **p** and **q** measured at locations **x**,**x'** and times t,t' but in our case these quantities are determined from the ensemble of spacecraft and not from single time series as is the case in any other method. The formulas for a continuous ensemble are given by /7/ We use in this paper the formulas for discrete REN{$\mathbf{q}_k$}, developed by /8/ constituting a modest modification of formulas from /7/. Applying the discrete Fourier convolution theorem, we calculate the single Fourier component of correlation functions after /7/ and /8/. For homogenous and stationary processes we calculate the cross-correlation function in Fourier space directly from quantities $\delta\mathbf{q}_k$, avoiding the difficult transformation of cross-correlation functions, especially for ensembles with a small number of systems (spacecraft) such as CLUSTER. After /7-9/ using the cross-correlation function for the mth single Fourier component we calculate the dimensionless helicity, the cross-helicity and proton compressibility.

APPLICATION OF THE MET TO THE RESULTS OF A 1D HYBRID SIMULATION OF UPSTREAM TURBULENCE IN FRONT OF QUASI-PARALLEL SHOCK

The application of the MET requires the availability of simultaneous multispacecraft data. For this report, we use magnetic field and density data with spatial and temporal resolutions of 4 ω_p/c and 1.6 Ω_p respectively, obtained from a 1D hybrid of a proton-electron plasma simulation of the upstream turbulence in front of a quasi-parallel shock (θ_{BN} = 20 deg). For adequate data aquisition we fly over the simulation box with the ensemble of 4 (synthetic) spacecraft with a velocity along x axis of about 3 V_A (in the downstream frame) toward the shock to produce a record of ensemble generated fluctuations of δB_y,δB_z ,δv_x, δv_y, δv_z and δn. From the simulation set up $\delta B_x = 0$ since $B_x = \cos(\theta_{BN})\, B_o$. All the fields are normalized to their upstream magnitude. An example of a time series obtained from an ensemble of systems is given in Figure 1.

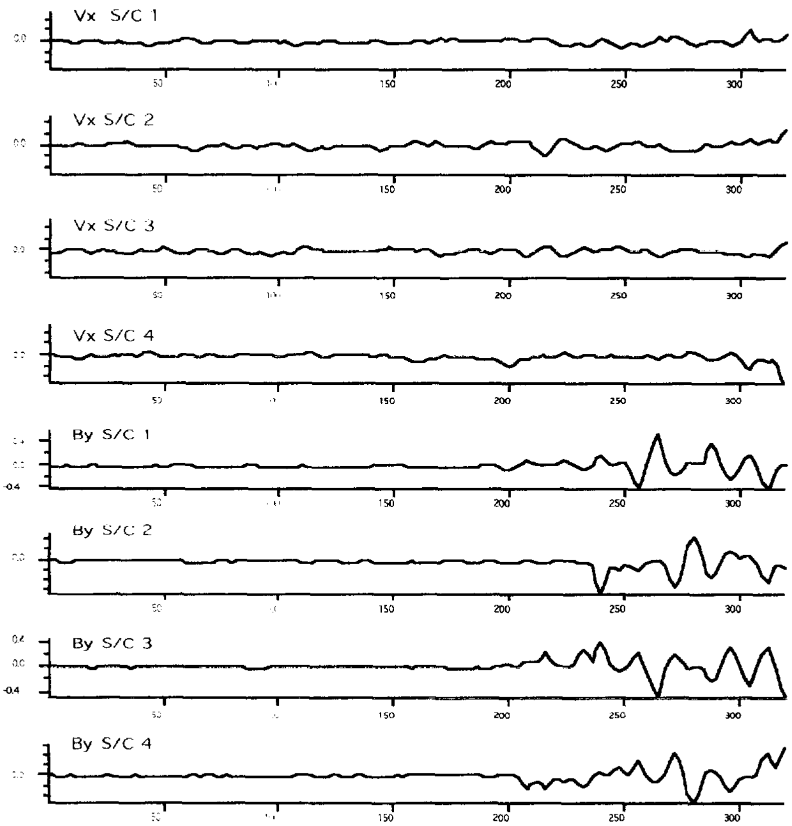

Figure 1. Simulation generated time record of the fluctuations of B_y and V_x calculated from equation 1 for each of 4 spacecraft (s/c) in ensemble.

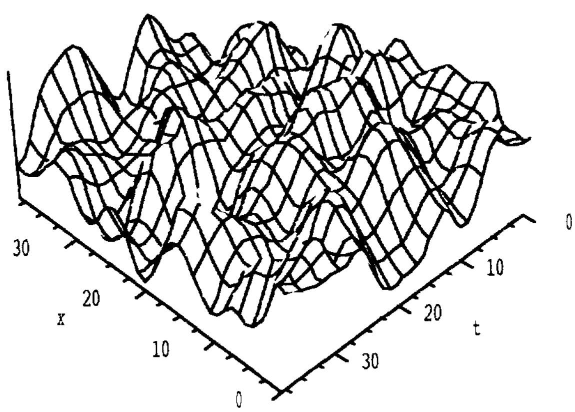

Figure 2. Example of the <A.b> correlation function. The x indicate position of the "ensemble" in the simulation frame in proton inertial length units and t indicate time lag in inverse proton gyrofrequency units. The location x, of the ensemble of system, to which all the averages are assigned, is determined by $\Sigma\ (x_k-x)^2$ = minimum where x_k is a location of kth system.

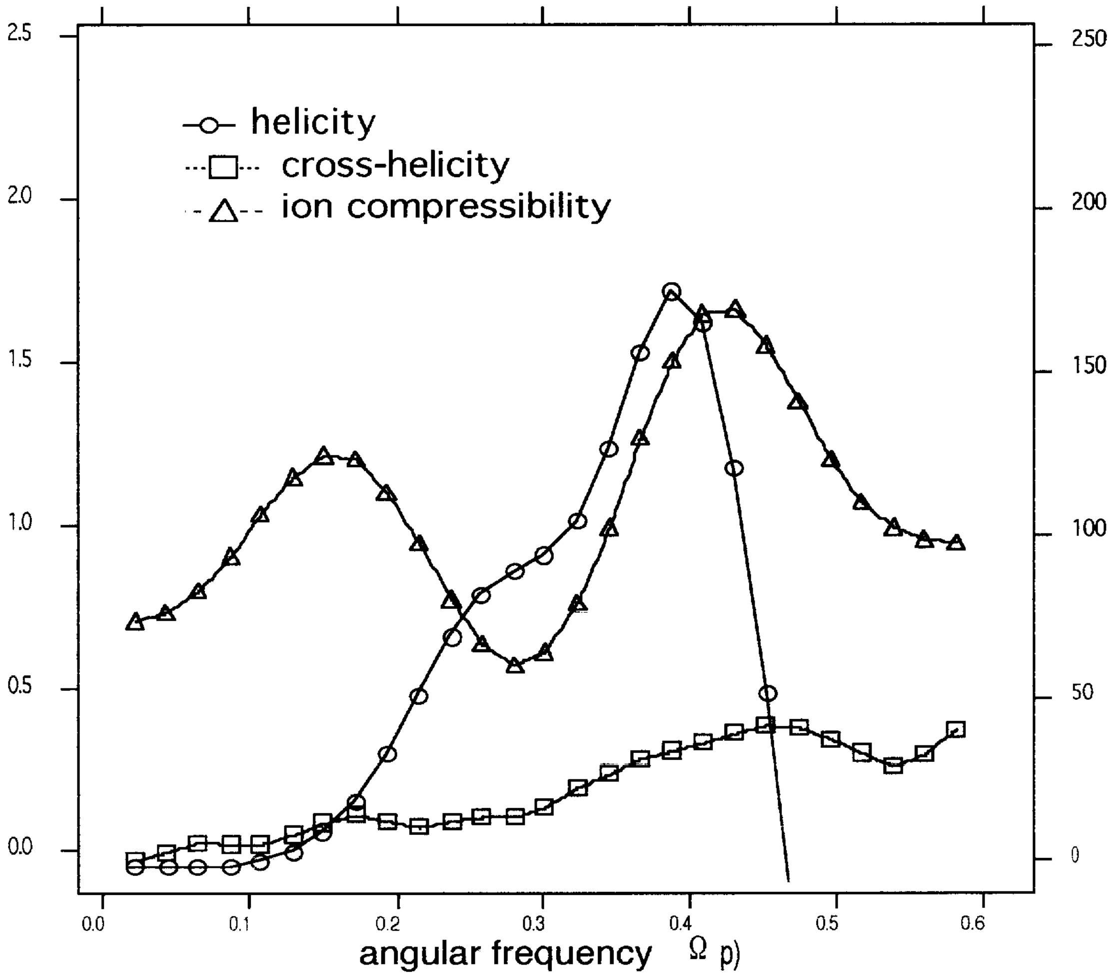

Figure 3. The helicity, cross-helicity (left scale) and proton compressibility (right scale) versus angular frequency of the turbulence in the downstream simulation frame calculated directly from 4 spacecraft records using the convolution theorem. /8/

Using trajectory data (here $x = V_{sc}t$), we calculate correlation functions <**δb**,**δb**>, <**δb**,**δA**> <**δv**,**δb**>and <**δb**,**δn**> in configuraton space . Figure 2 shows an example of the <**δb**,**δA**> correlation as a function of location x, in the simulation frame in proton inertial length units and a time lag in the inverse proton gyrofrequency units respectively. These correlations represent and describe synthetized average properties of the fluctuating fields of the simulated upstream turbulence.

The spatio-temporal analysis of these functions requires their decomposition into elementary structures (generalized functions) . Such a decomposition can be done more generally using the so called wavelet transform /10/,/11/ and /12/. Assuming periodicity of the correlation functions we here use the Fourier transform to obtain the transformed functions and interpret their appropriate ratios as helicity, cross-helicity and compressibility of the investigated upstream turbulence.

For the frequencies in the simulation frame exceeding 0.01 Ω_p we assume that we can sufficiently well approximate the average (expected) values of helicity, cross-helicity and and compressibility in Fourier space.

The Fourier spectral analysis (not shown) indicates that the enhanced fluctuations observed are predominantly between 0.3 and 0.4 Ω_p in the downstream frame. Figure 3 shows, helicity, cross-helicity and and proton compressibility within this range.
The positive value of helicity (from 0.5 to 2) indicates elliptical polarization. The high compressibility of about 60-170 (very strongly compressional fluctuation) and small cross-helicity values < 0.5 clearly indicate that the analysed fluctuations have average properties generally consistent (within a scale factor) with the linear FMS kinetic mode directly calculated from linear Vlasov theory /13/.

SUMMARY

In this report we introduced and applied a different, inherently multi-point processing technique of the data returned from an group of spacecraft, the multi-spacecraft ensemble technique, the MET. We note that many other contemporary data analysis and interpretation techniques using single time series of measurements are based on the assumption of stationarity and/or linearity of the plasma process. These methods rely on the notion that observed field and/or plasma fluctuations can be approximated by one planar wave or linear superpositions of many independent planar waves with amplitudes determined by the deviation of the measured quantity from the mean time average over arbitrarily selected time interval. Such an arbitrarily selected time interval is perceived (and assumed) to be much longer than the spatial scale of investigated fluctuations. Such an approach while in some cases justified by the nature of the fluctuations, is in general methodically unsound and often results in large errors in the amplitude and frequency estimations especially in application to fully developed turbulence. A turbulent process involves not only rapid changes in the amplitudes of fluctuating fields but also may involve the time dependent transfer of wave power from one to another region of the frequency spectrum, the so called turbulent cascading. For example such an effect occurs in the non-linear stage of the development of the initially linearly growing well known fire-hose instability. In these turbulent conditions commonly found in space plasma other data analysis methods described above often are methodically inadequate.
In contrast, the MET technique is more general that other techniques in the sense that requires no assumptions about stationarity and linearity. However, the MET still requires strict compliance with condition 2 and 3. We showed in the report that the MET, while methodically difficult, can be succesfully applied to the analysis of numerical simulation data of the collisionless shock and upstream turbulence. The resulting values of helicity cross-helicity and compressibility obtained from the MET are generally consistent with those obtained from kinetic calculations.
The MET represent a qualitatively different approach to the multi-spacecraft data analysis and challenge in the data interpretation. The application of the MET to the CLUSTER mission will require that in a given configuration the CLUSTER of spacecraft form a representative ensemble. This can be easily accomplished in the upstream region, for example where spatial and temporal scales of the electromagnetic, plasma proccesses, obtained using two-spacecraft technique /1/ have already been studied . Moreover, for the CLUSTER mission significant improvements in the dynamic range, sensitivity, signal/noise ratio, temporal and angular resolution were made in a new generation of instruments which will satisfy conditions 2 and 3 for a variety of magnetospheric processes and thus can be effectively investigated using the multispacecraft ensemble technique (MET).

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