Introduction to Coherence, Polarization and Ellipticity: Background for Problem Sets 2 and 3

ESS 265

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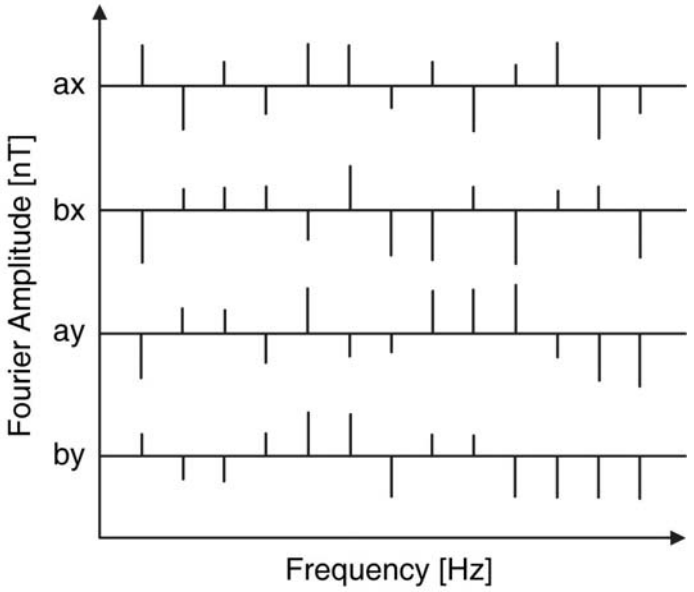
Coherence

- To determine the coherence of two signals x_i ; y_i, first find Fourier amplitudes a_{xi}, b_{xi} ; a_{yi}, b_{yi}



- Then the coherence = $\sum_i (a_{xi}a_{yi} + b_{xi}b_{yi}) / [\sum_i (a_{xi}^2 + b_{xi}^2)]^{1/2} [\sum_i (a_{yi}^2 + b_{yi}^2)]^{1/2}$
- If $ay_i = kax_i$ and $by_i = kbx_i$, then coherence = 1

Determining Percent Polarization and Ellipticity

- Calculate Cospectrum over spectral band

$$\begin{pmatrix} \sum_i (a_{xi}^2 + b_{xi}^2) & \sum_i (a_{xi}a_{yi} + b_{xi}b_{yi}) & \sum_i (a_{xi}a_{zi} + b_{xi}b_{zi}) \\ \sum_i (a_{xi}a_{yi} + b_{xi}b_{yi}) & \sum_i (a_{yi}^2 + b_{yi}^2) & \sum_i (a_{yi}a_{zi} + b_{yi}b_{zi}) \\ \sum_i (a_{xi}a_{zi} + b_{xi}b_{zi}) & \sum_i (a_{xi}a_{zi} + b_{xi}b_{zi}) & \sum_i (a_{zi}^2 + b_{zi}^2) \end{pmatrix}$$

- Find the eigen values and eigen vectors of the real symmetric matrix
- Use these to diagonalize matrix $\lambda_1^2, \lambda_2^2, \lambda_3^2$ are the maximum, intermediate, and minimum eigen values. Assume minimum eigen value is the noise

$$\begin{pmatrix} \lambda_1^2 & 0 & 0 \\ 0 & \lambda_2^2 & 0 \\ 0 & 0 & \lambda_3^2 \end{pmatrix}$$

- Percent Polarization is

$$1 - \lambda_3^2 / (\lambda_1^2 + \lambda_2^2)$$

- Eccentricity is

$$\lambda_2 / \lambda_1$$