ESS 265
Introduction to XTRANS and Coordinate Transformation in Conjunction with Problem Set 4

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Geophysical Coordinate Transformations

- Many different coordinate systems are used to define measurement and observer locations both on Earth and in space.
- Generally, there is a coordinate system that simplifies some aspect of the problem, such as making a variation 2-dimensional.
- It can be very tricky to transform from one system to another, especially when angles and spherical trigonometry are involved.
- Vector-matrix formulation can simplify this process. The XTRANS program is designed to help the user by providing many of these transformations in use in solar terrestrial physics.
- Details of coordinate systems and their usage can be found in Appendix 3 of Kivelson and Russell.

Simple Rules of Coordinate Transformation

- If the *matrix A* transforms *vector* V^a measured in *system a* to V^b measured in *system b*, then its transpose A^T transforms V^b into V^a

$$A \cdot V^a = V^b$$

$$A^T \cdot V^b = V^a$$

- To construct the transformation *matrix A*, find the direction cosines of the three new coordinate axes for *system b* in *system a* coordinates. Let these vectors be (x_1, x_2, x_3), (y_1, y_2, y_3) and (z_1, z_2, z_3). Then

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix} \begin{pmatrix} v_x^a \\ v_y^a \\ v_z^a \end{pmatrix} = \begin{pmatrix} v_x^b \\ v_y^b \\ v_z^b \end{pmatrix}$$

and

$$\begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \begin{pmatrix} v_x^b \\ v_y^b \\ v_z^b \end{pmatrix} = \begin{pmatrix} v_x^a \\ v_y^a \\ v_z^a \end{pmatrix}$$

Commonly Used Coordinate Systems

- Geocentric Equatorial Inertial (GEI)
  - X axis to the first point of Aries (direction of Sun to Earth about September 21)
  - Z axis parallel to Earth's rotation axis
  - Y = Z x X

  Used for spacecraft orbit calculations

- Geographic Coordinate System (GEO)
  - X axis from center of Earth to Greenwich meridian in equatorial plane
  - Z axis parallel to rotation axis of Earth
  - Y = Z x X

  Used for terrestrial maps

- GEO differs from GEI by a rotation about Z dependent on sidereal not universal (solar) time (UT).

- Universal Time is defined as 12h minus the longitude of the Sun converted to degrees by dividing by 15.

Geophysical Coordinates

- Geomagnetic coordinates (MAG) have Z along the magnetic dipole axis northward. The Y axis is perpendicular to the plane containing the rotational poles and the dipole. This coordinate system has a fixed transformation from GEO to the extent that the Earth's internal field is steady. This coordinate system is useful for geomagnetic studies at and near the surface of the Earth.

- Geocentric Solar Ecliptic (GSE) has its X axis to the Sun and Z along the north pole of the Earth's orbit.

- Geocentric Solar Equatorial (GSEQ) has its X axis to the Sun with the Sun's rotational axis in the X-Z plane (northward). It orders interplanetary magnetic field data.

- Geocentric Solar Magnetospheric (GSM) has its X axis to the Sun with the Earth's dipole in the X-Z plane. This is used to order observations near the magnetopause and in the magnetotail.

- Solar Magnetic (SM) has its Z axis along the dipole axis and the X-Z plane contains the directions to the Sun. It is used to order measurements in the inner magnetosphere. Magnetic local time is defined in this coordinate system.

Other Coordinate Systems

- Many coordinate systems are defined by the location of the observer.

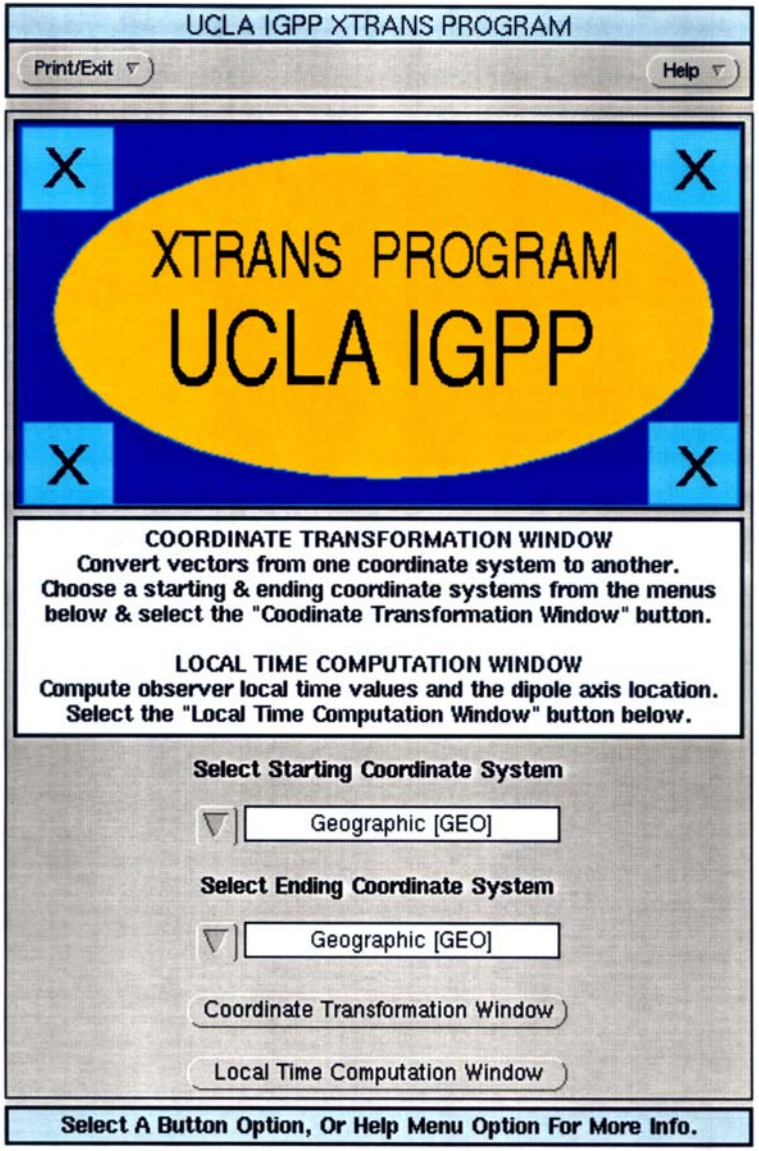
- Some coordinates depend on the orientation of a boundary normal
  - Shock normals
  - Magnetopause normals

  These coordinate systems can be derived from models or data.

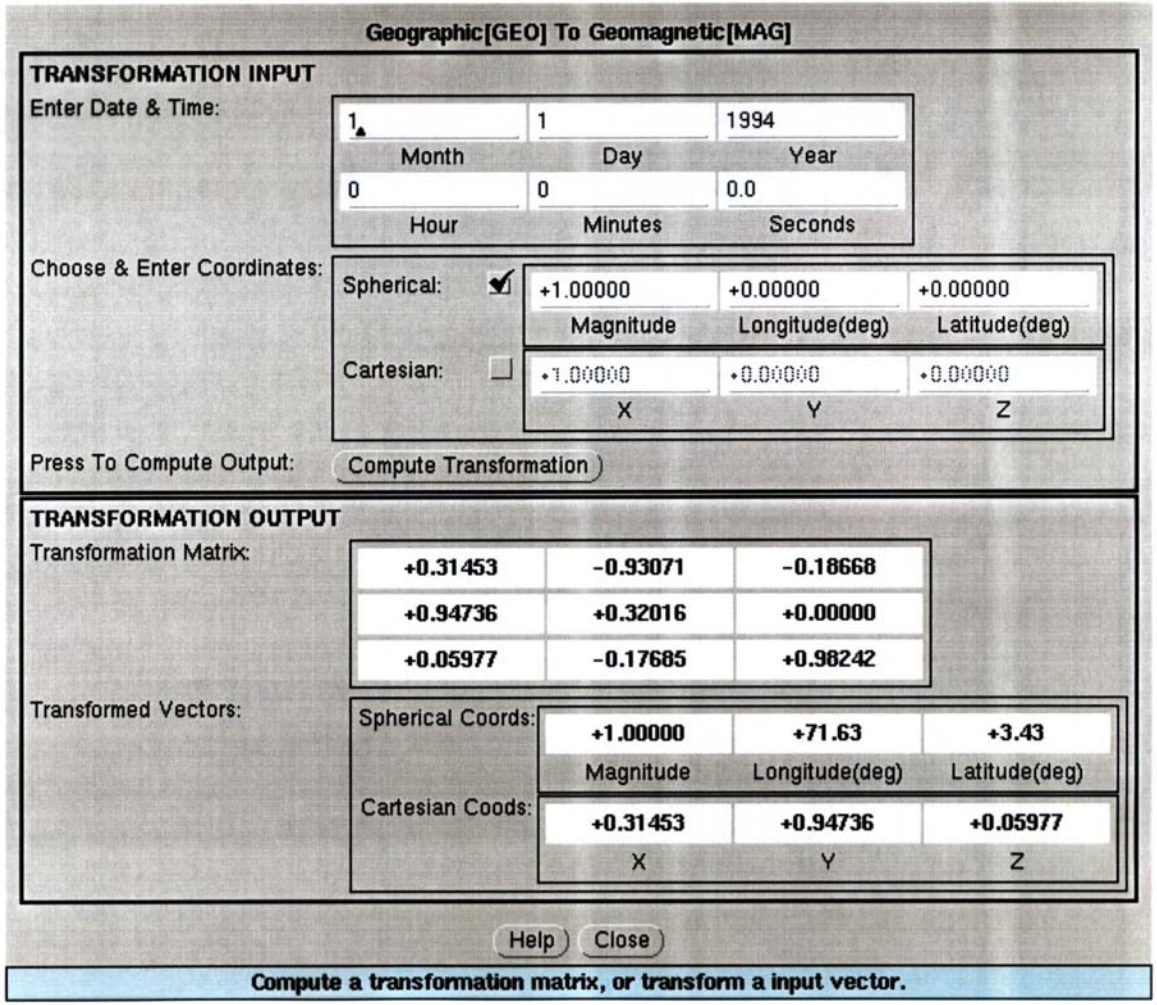
- The study of waves particularly benefits from the use of the principal axis system.

XTRANS Program

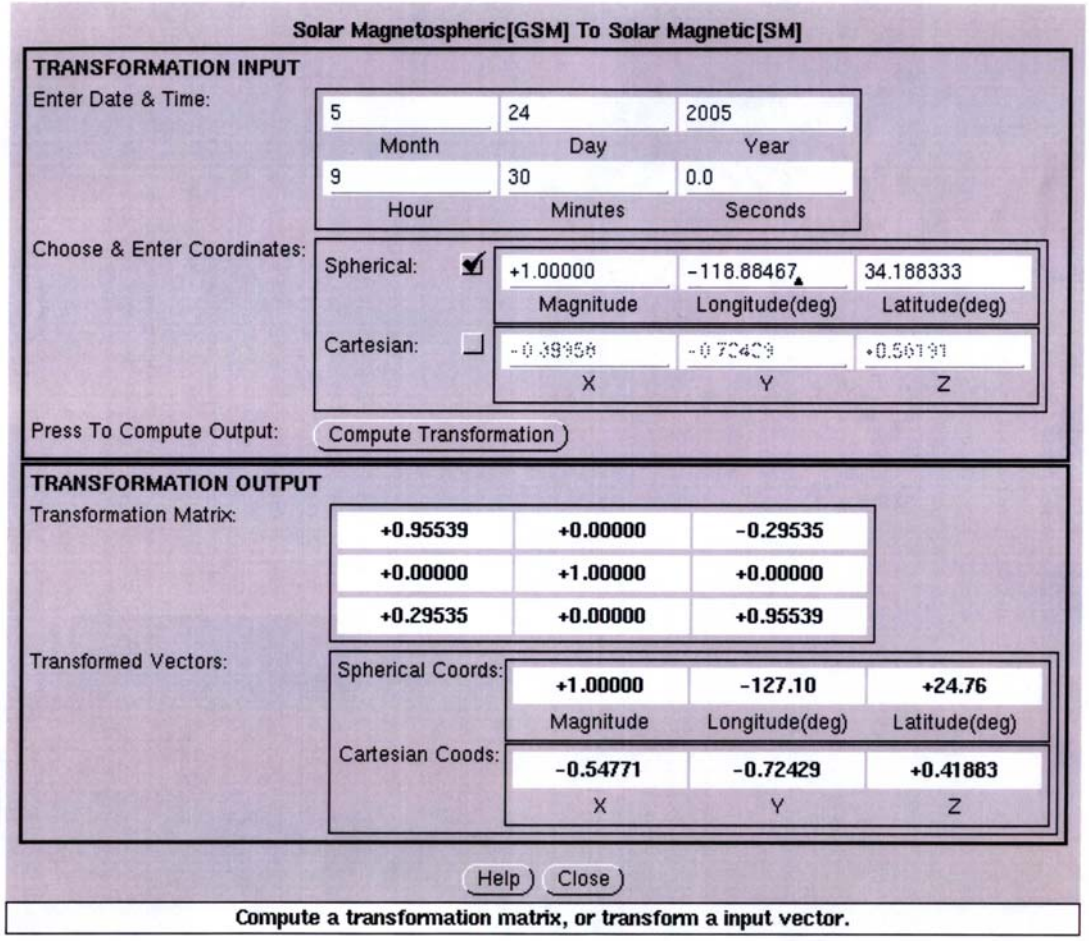
- XTRANS can transform vectors into other coordinate systems, even those that are time-dependent.
- It works in either cartesian or spherical coordinates.
- It provides the transformation matrix.
- It can also determine local time and local magnetic time.



Geographic to Geomagnetic



GSM to SM


Solar Magnetospheric[GSM] To Solar Magnetic[SM]
TRANSFORMATION INPUT
Enter Date & Time:
5
Month
24
Day
2005
Year
9
Hour
30
Minutes
0.0
Seconds
Choose & Enter Coordinates:
Spherical:
+1.00000
Magnitude
-118.88467
Longitude(deg)
34.188333
Latitude(deg)
Cartesian:
-0.39958
X
-0.72429
Y
+0.56191
Z
Press To Compute Output:
Compute Transformation
TRANSFORMATION OUTPUT
Transformation Matrix:
+0.95539
+0.00000
-0.29535
+0.00000
+1.00000
+0.00000
+0.29535
+0.00000
+0.95539
Transformed Vectors:
Spherical Coords:
+1.00000
Magnitude
-127.10
Longitude(deg)
+24.76
Latitude(deg)
Cartesian Coods:
-0.54771
X
-0.72429
Y
+0.41883
Z
Help
Close
Compute a transformation matrix, or transform a input vector.

Local Time and Dipole Axis