ESS 265: Instrumentation, Data Processing and Data Analysis in Space Physics

Lecture 16: Alternate Approaches to Time Series Analysis

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Problems with Fourier Analysis

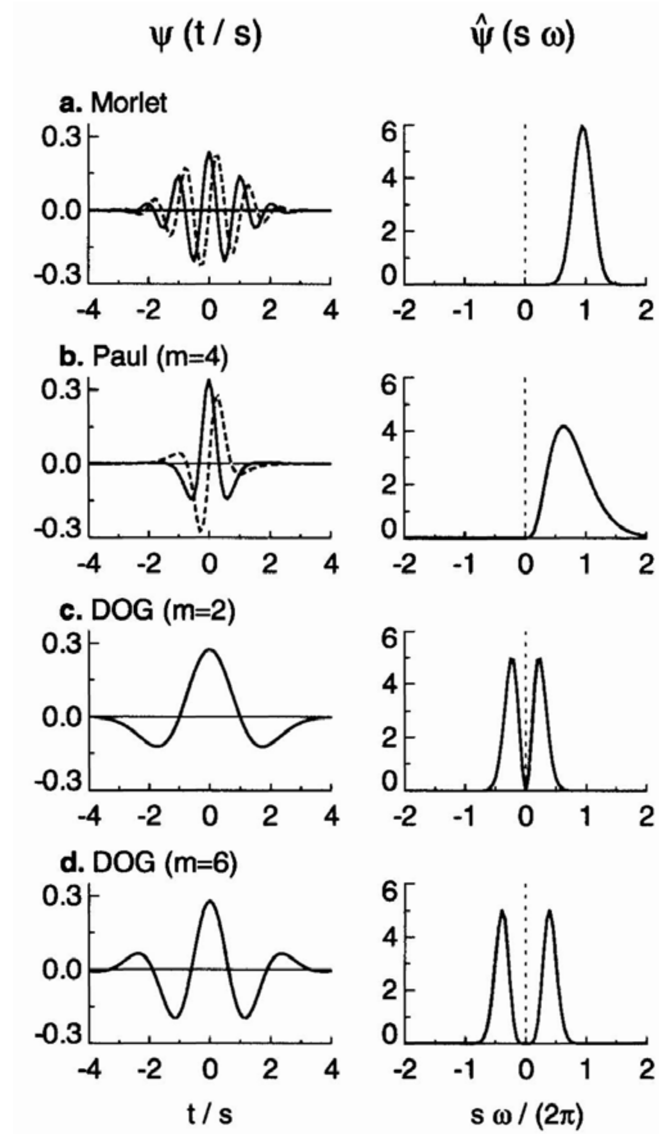
- Fourier analysis has been a standard analysis tool for over half a century and is well understood, but it has some limitations.
  - It assumes that a time series repeats forever
  - It uses the same analysis interval for low frequencies and high
  - It uses an analyzing function that may not at all look like the signal to be analyzed

- Two alternate and very different approaches are:
  - Wavelets – they provide many different possible analyzing functions
  - Empirical Mode Decomposition – this allows the time series to be decomposed into a series of its natural waveforms.

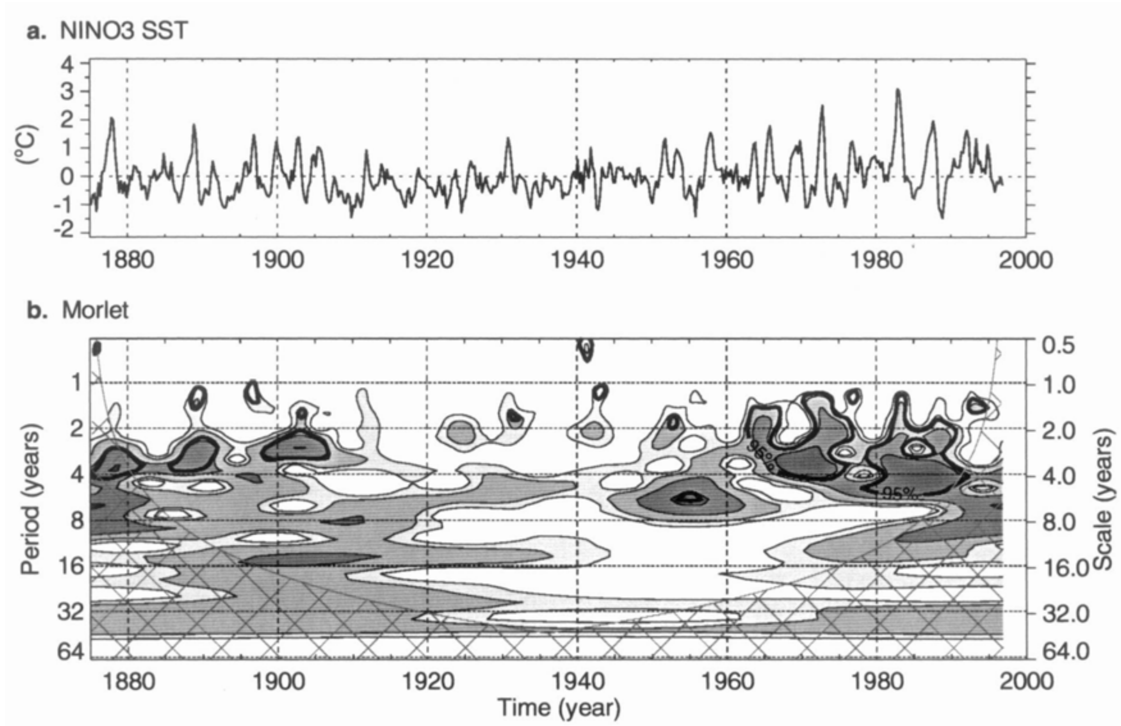
Wavelet Analyzing Functions

- Fourier analysis is performed by finding the amplitude of sine and cosine waves that can reproduce a time series. In dynamic spectral analysis, we use short segments of sines and cosines (but remember our infinitely repeating assumption).

- Wavelet-analyzing functions are spatially limited functions as shown here. If they have a real and imaginary form, i.e. in phase and quadrature forms that are orthogonal, then we can calculate all the wave-analysis techniques we have been using: phase difference, coherence, direction of propagation, ellipticity.

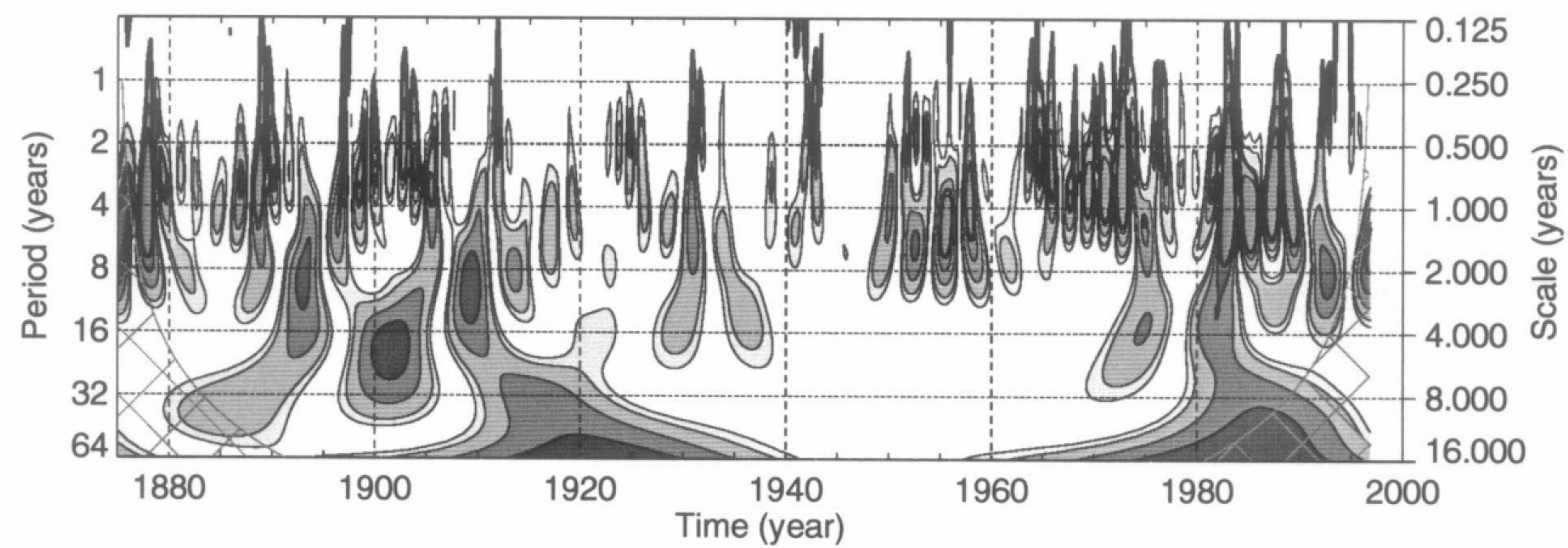


An Application 1



- Here is a time series of the sea surface temperature in the El Nino producing zone of the ocean from 1875 to 1997.
- The bottom panel is a periodogram (dynamic spectrum) covering variations with periods from 0.5 to 64 years, showing how the energy in these period ranges varied over the length of the record using a Morlet wavelet.
- Contours drawn at normalized variances of 1, 2, 5, 10 σ^2 where $\sigma = 0.7°$ C. Thick contour encloses regions of greater than 95% confidence. Cross-hatched region shows "cone of influence" where edge effects are important.

An Application 2



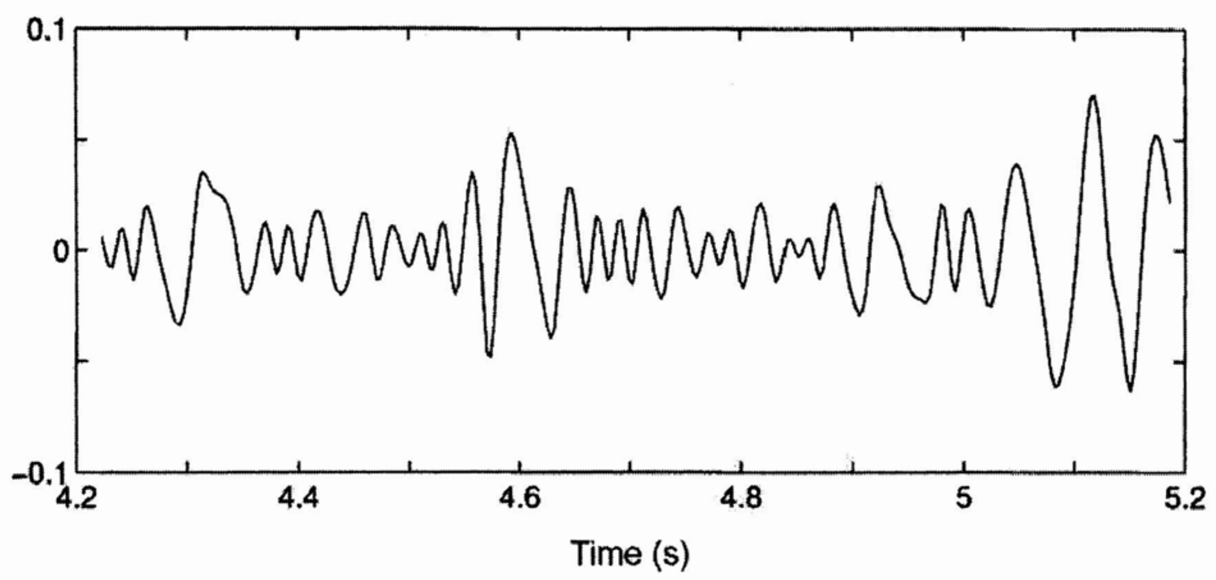
- Here the analysis is repeated using a derivative of a Gaussian wavelet (DOG, m = 2), also called a Mexican hat wavelet. Since this is a real (not complex) wavelet, it cannot be used to calculate phase for example.
- This function localizes the signals better at all frequencies, but spreads the power in period (frequency). Clearly there is an uncertainty principle at work here. You have a certain product of frequency and time resolution you can achieve, and you can sacrifice one for the other by choosing a different wavelet type.

The Equations Behind the Wavelets

Name	$\psi_0(\eta)$	$\hat{\psi}_0(s\omega)$	e-folding time τ_s	Fourier Wavelength λ
Morlet (ω_0 = frequency)	$\pi^{-1/4} e^{i\omega_0\eta} e^{-\eta^2/2}$	$\pi^{-1/4} H(\omega) e^{-(s\omega-\omega_0)^2/2}$	$\sqrt{2}\,s$	$\frac{4\pi s}{\omega_0+\sqrt{2+\omega_0^2}}$
Paul (m =order)	$\frac{2^m i^m m!}{\sqrt{\pi(2m)!}}(1-i\eta)^{-(m+1)}$	$\frac{2^m}{\sqrt{m(2m-1)!}} H(\omega)(s\omega)^m e^{-s\omega}$	$s/\sqrt{2}$	$\frac{4\pi s}{2m+1}$
DOG (m =derivative)	$\frac{(-1)^{m+1}}{\sqrt{\Gamma(m+\frac{1}{2})}} \frac{d^m}{d\eta^m}\left(e^{-\eta^2/2}\right)$	$\frac{i^m}{\sqrt{\Gamma(m+\frac{1}{2})}}(s\omega)^m e^{-(s\omega)^2/2}$	$\sqrt{2}\,s$	$\frac{2\pi s}{\sqrt{m+\frac{1}{2}}}$

- Wavelets have found wide application in the communication industry and in image data compression, so there is an extensive literature on them.
- Here we show several wavelets and their Fourier transforms. The Fourier transforms allow shortcuts in the computation of the wavelet coefficients by the convolution theorem. [The wavelet transform is the inverse Fourier transform of the convolution of the Fourier Transform of the time series and the Fourier transform of the wavelet.] Thus, in a sense, wavelet analysis rides on top of Fourier analysis.

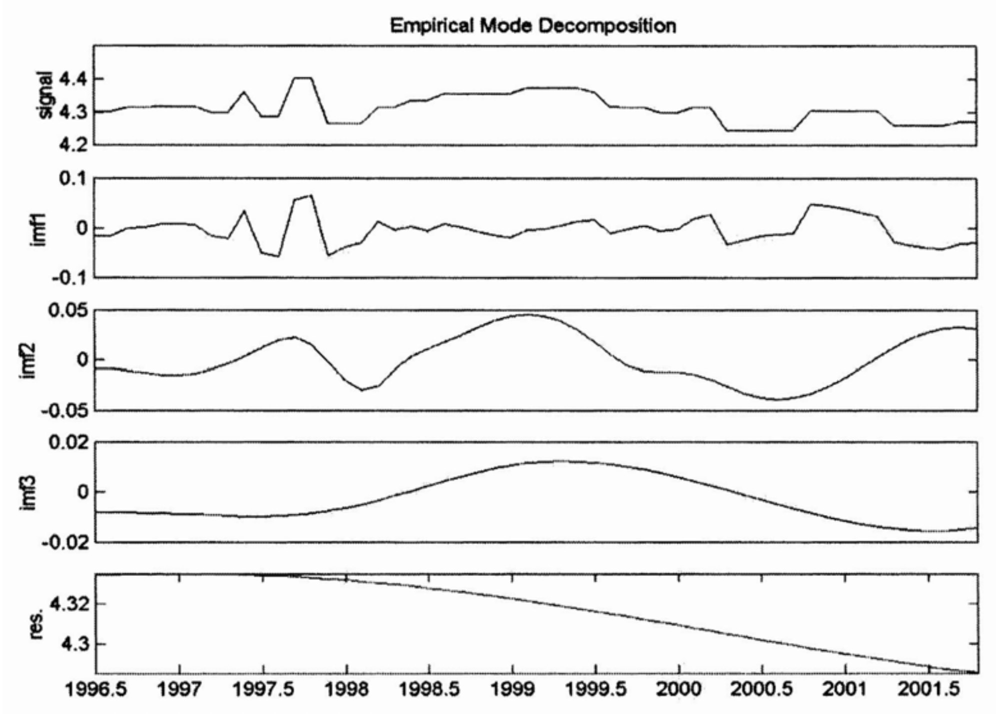
Hilbert-Huang Transform: Empirical Mode Decomposition



- For analyzing non-linear, non-stationary data.
- Data is decomposed (sifted) into a finite (usually small) number of “intrinsic mode functions” (IMF).
- Method is adaptive and highly efficient.
- Using the Hilbert transform, the IMFs can give instantaneous frequencies.
- An IMF satisfies two conditions:
  – In the whole data set, the number of extrema and the number of zero crossings must be equal or differ at most by one.
  – At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.
- Above is an IMF satisfying these conditions.

An Application: Time Variation of the Jovian Field

- The sifting process proceeds as follows:
  - The signal has at least two extrema: one maximum and one minimum.
  - The characteristic time scale is defined by the time lapse between extrema.
  - If no extrema, but only inflection points, then it can be differentiated to reveal extrema.

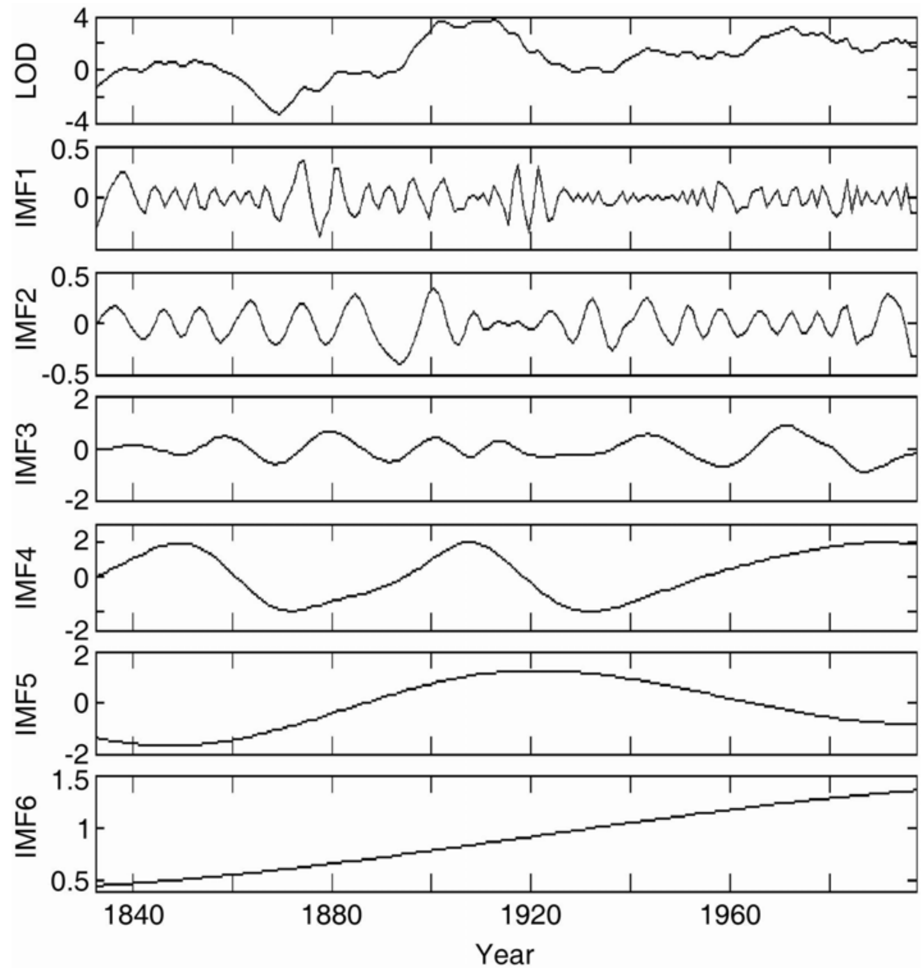


- Here we look at the jovian magnetic moment to see how it varied over the lifetime of the Galileo mission.

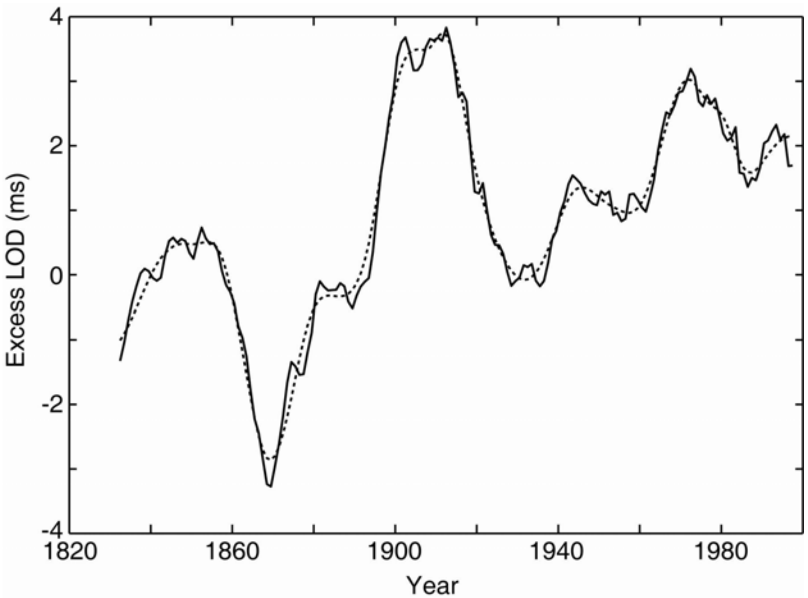
- Here we are most interested in the bottom IMF, the trend.

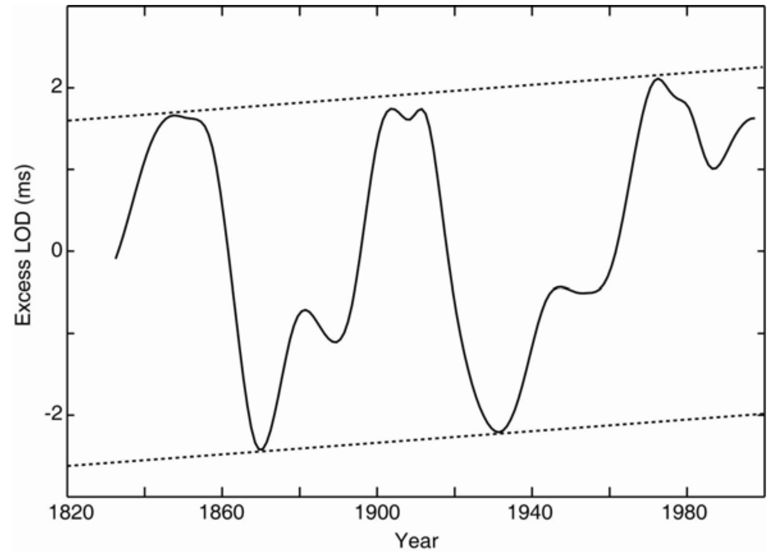
An Application: The Length of the Day

- While everyone feels that time is going by more quickly these days, in fact days are (slowly) getting longer.
- The secular change is a combination of tidal effects and glacial rebound.
- Lots of effects change the length of the day: heating of the atmosphere, ocean currents, winds, etc.
- There has been a report of a 60-year variation. Is it real? If so, what is its cause?
- Here is the LoD from 1832 to 1997 sifted into IMFs.
- This work has been reported by Roberts et al., Geophysical and Astrophysical Fluid Dynamics, 101, 11-35, 2007.

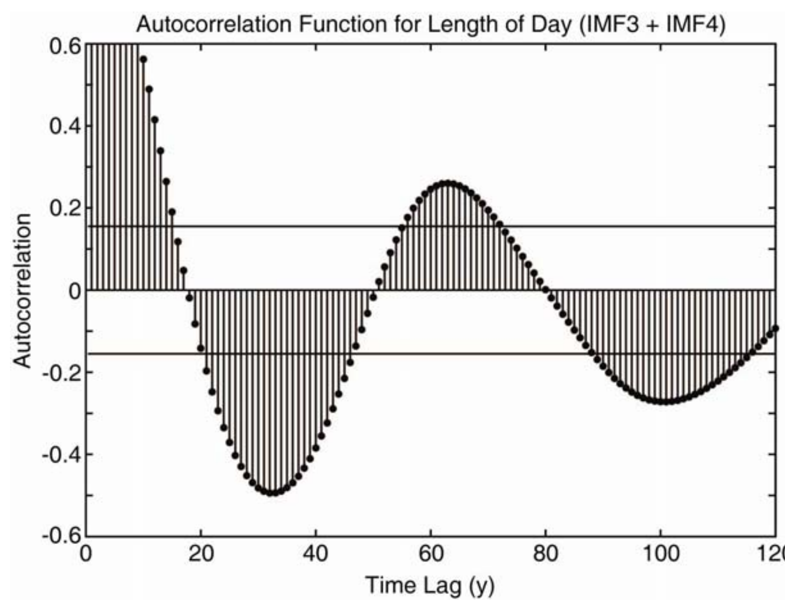


Finding the 60-year Variation





- By comparing the original time series with the time series with IMF1 and IMF2 removed, we prove these two IMFs have little effect on the time series.
- If we examine by eye the sum of IMF3 and 4, we can see a 60-year variation.
- If we calculate the autocorrelation function of this time series, we see a peak at near 60 years.



Coupling with the Geomagnetic Field

- One pendulum swinging in the Earth that would have a 60-year period is the core. It could rotate back and forth relative to the crust and mantle connected by the magnetic field.

- To test this hypothesis, Roberts et al. searched for the oldest geomagnetic records and analyzed them in the same way.

- We found many problems in the data, but there was a global effect in the measurements of declination seen at many stations with similar phasing.

- We concluded that the inner core is coupled to the mantle and crust by the magnetic field, and the system oscillates like a torsion pendulum at a 60-year period.