The Physics of Field-Aligned Currents

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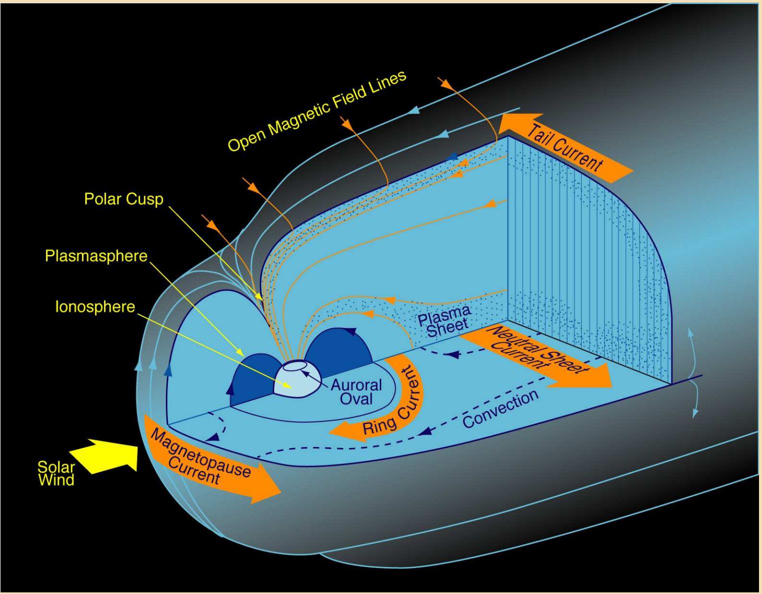
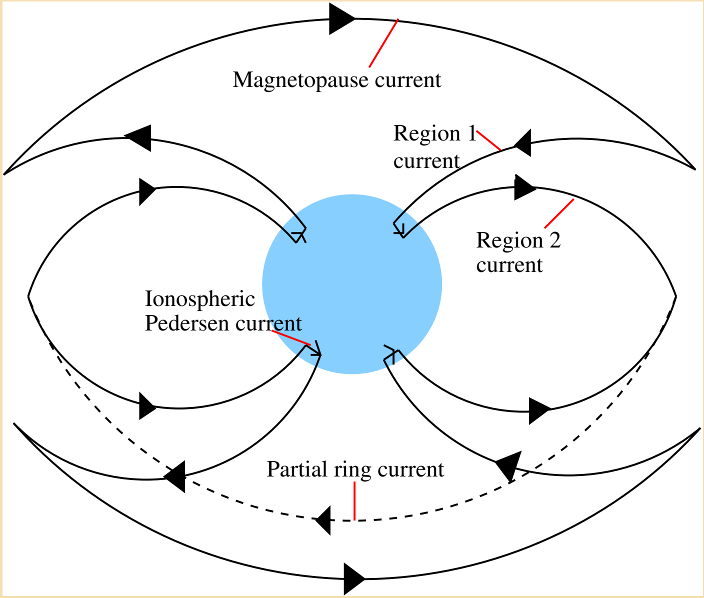
Magnetospheric Current Circuit

There is a rich structure of currents flowing parallel $(j_\parallel)$ and perpendicular $(\mathbf{j}_\perp)$ to the magnetic field $(\mathbf{B})$.

- Magnetopause current
- Ring current
- Ionospheric currents
- Region 1 and 2 currents
- ULF Alfvén waves



[Figure from NASA GSFC.]



View from tail

Quasi-Neutrality and Current Continuity

- The Ampère-Maxwell equation states

$$\nabla \times \mathbf{B}/\mu_0 = \mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

- Taking the divergence yields

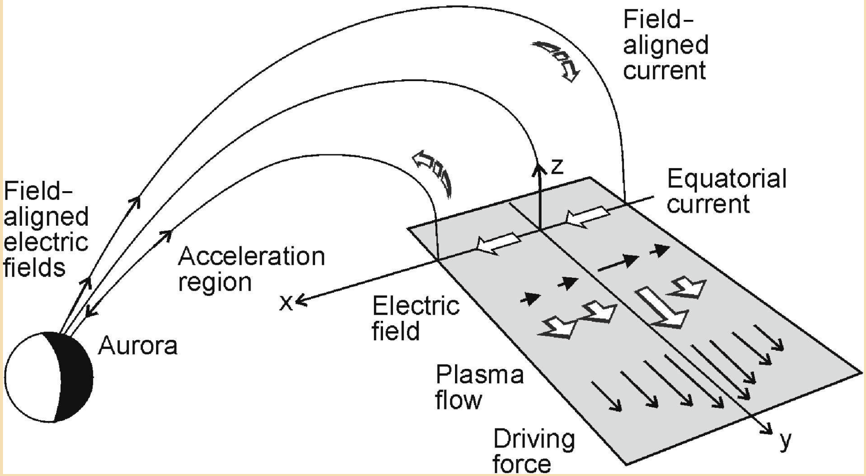
$$0 = \nabla \cdot \mathbf{j} + \varepsilon_0 \frac{\partial \nabla \cdot \mathbf{E}}{\partial t}.$$

- Using Gauss's Law, $\nabla \cdot \mathbf{E} = \rho^*/\varepsilon_0$ (where ρ^* = net charge density) we find

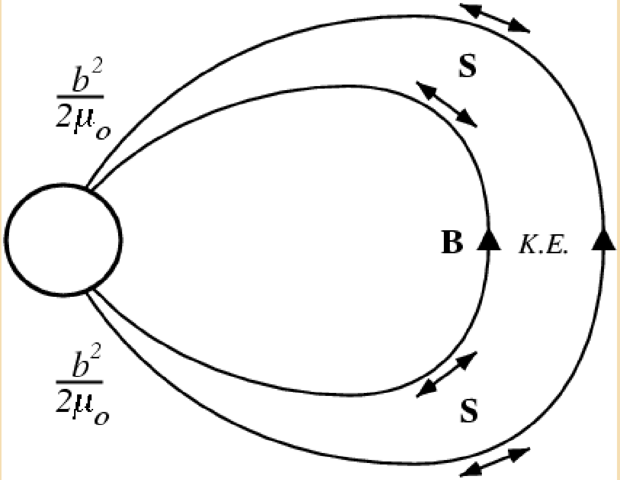
$$\nabla \cdot \mathbf{j} + \frac{\partial \rho^*}{\partial t} = 0.$$

- Quasi-neutrality $(\rho^* \approx 0) \Rightarrow$
  - $\nabla \cdot \mathbf{j} \approx 0$: Currents are self-closing.
  - The displacement current ($\varepsilon_0 \partial \mathbf{E}/\partial t$) is neglected.
  - $\rho^* \approx 0$ means $(n_e - n_i)/n_e \ll 1$.
  - Debye length is $\sqrt{\varepsilon_0 k T_e/e^2 n_e} \sim$ 100 m (magnetosphere), 0.01 m (ionosphere).

Some Magnetohydrodynamic Driving Mechanisms



- Imposed velocity shear [*Rönnmark*, GRL, 1998]



- Fundamental standing (ULF) Alfvén wave
- Axisymmetric oscillation of magnetic shells
- b_ϕ and u_ϕ components only
- $j_\parallel$ and $j_\perp$ currents

Generation of $j_\|$ (MHD description)

- Single fluid MHD momentum equation

$$\rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{j}\times\mathbf{B} - \boldsymbol{\nabla} p + \mathbf{F}$$

- Solve for $\mathbf{j}_\perp$

$$\mathbf{j}_\perp = -\left(\rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + \boldsymbol{\nabla} p - \mathbf{F}\right)\times\frac{\mathbf{B}}{B^2}$$

- Since $\boldsymbol{\nabla}\cdot\mathbf{j} = 0$ we find $j_\|$

$$\frac{\partial j_\|}{\partial s} + \boldsymbol{\nabla}_\perp\cdot\mathbf{j}_\perp = 0, \qquad \Rightarrow \qquad j_\| = -\int_{\text{along }\mathbf{B}} \boldsymbol{\nabla}_\perp\cdot\mathbf{j}_\perp \mathrm{d}s$$

- The field-aligned current ($j_\|$) flows to maintain quasi-neutrality $(\rho^* \approx 0)$

What is $j_\parallel$ microscopically?

- Net drift of charged particles parallel to **B**
- Consider the parallel component of the equation of motion for a charged particle (uniform **B**),

  $$\frac{\mathrm{d}v_\parallel}{\mathrm{dt}} = \frac{qE_\parallel}{m}.$$

  $E_\parallel$ can be used to establish a field-aligned current.
- Since $m_e/m_i \ll 1$ the electrons are more mobile, and carry most of the parallel current: $j_\parallel \approx \sum -ev_{e\parallel}$
- In MHD $m_e/m_i \to 0$: electrons are represented by a massless charge-neutralizing fluid ($E_\parallel \to 0$)
- To generate $j_\parallel$ requires $E_\parallel$
  - ❍ How does $E_\parallel$ arise?
  - ❍ Causal interpretation in ideal MHD difficult since $\partial\mathbf{E}/\partial t$ has been neglected

Guiding centre description of particle motion

- Particles execute a circular trajectory about **B** whose centre drifts. Valid when
  - Gyroradius $\ll$ background scale length
  - Gyroperiod $\ll$ background timescale
- Guiding centre drifts parallel to **B** arise from
  - $E_\parallel$ and magnetic mirror force

$$m\frac{\mathrm{d}v_\parallel}{\mathrm{d}t} = qE_\parallel + \mu\nabla_\parallel B$$

  - The magnetic moment $\mu = mv_\perp^2/2B$ is a constant of motion.
- Guiding centre drifts perpendicular to **B** (can depend upon q, m and energy) arise from
  - Grad B drift
  - B curvature drift
  - Polarization drift (**E**(t))
  - **E** × **B** drift

Generation of $j_\parallel$ (particle description)

- In general, given $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$, electrons and ions drift relative to one another $\Rightarrow$
    - current can flow
    - net charge density is likely to develop $\rho^* \neq 0$
- As ρ^* becomes non-zero, $E_\parallel$ and $\mathbf{E}_\perp$ change to satisfy

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \rho^* / \varepsilon_0$$

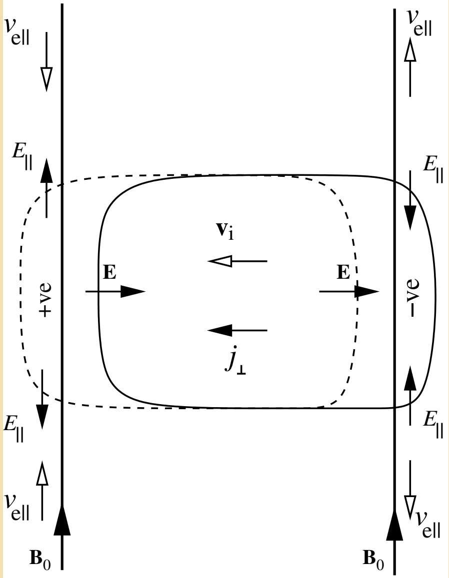
  and influence the particle motion

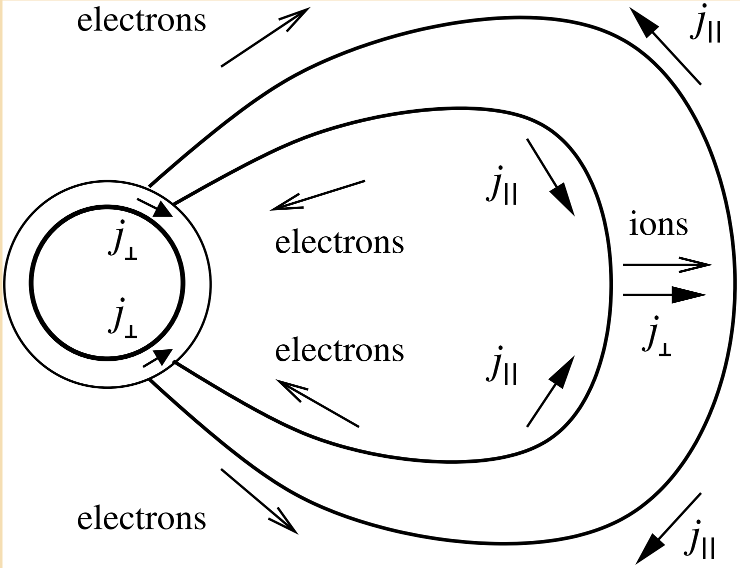
- Effect of even a small $E_\parallel$:
    - electrons are accelerated parallel to **B**
    - $j_\parallel$ is established
    - parallel electron motion acts to reduce ρ^*
    - the plasma remains quasi-neutral ($\rho^* \approx 0$)
- Interestingly, **E** still satisfies

$$\boldsymbol{\nabla} \times \mathbf{B}/\mu_0 = \mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t},$$

  even though the last term is still negligible.

Ring Current and Standing Alfvén Wave $j_\parallel$



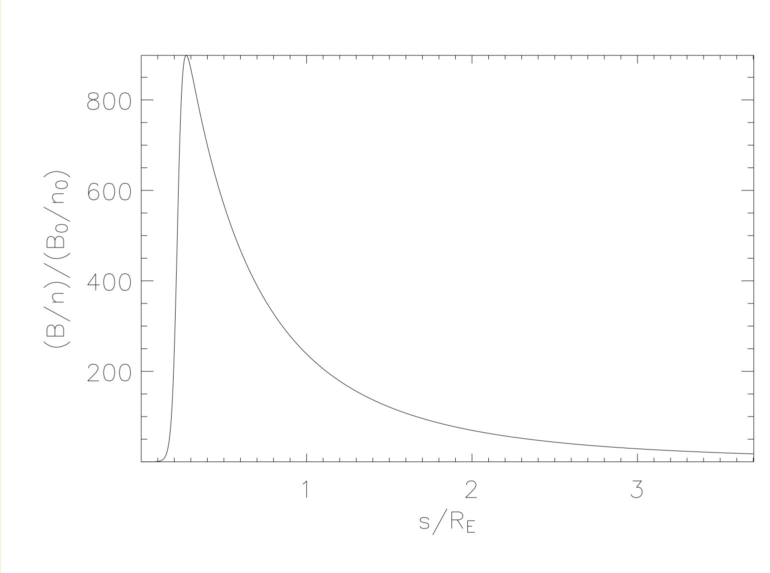


- Looking earthward from the tail
- Warm ion cloud drifts westward
- Slight charge imbalance generates **E**, electron motion and $j_\parallel$

- Snapshot of Alfvén wave current circuit
- $\mathbf{E}(t)$ in equatorial region produces polarization drift
- Ions drift across L-shells leading to charge imbalance and subsequent $j_\parallel$

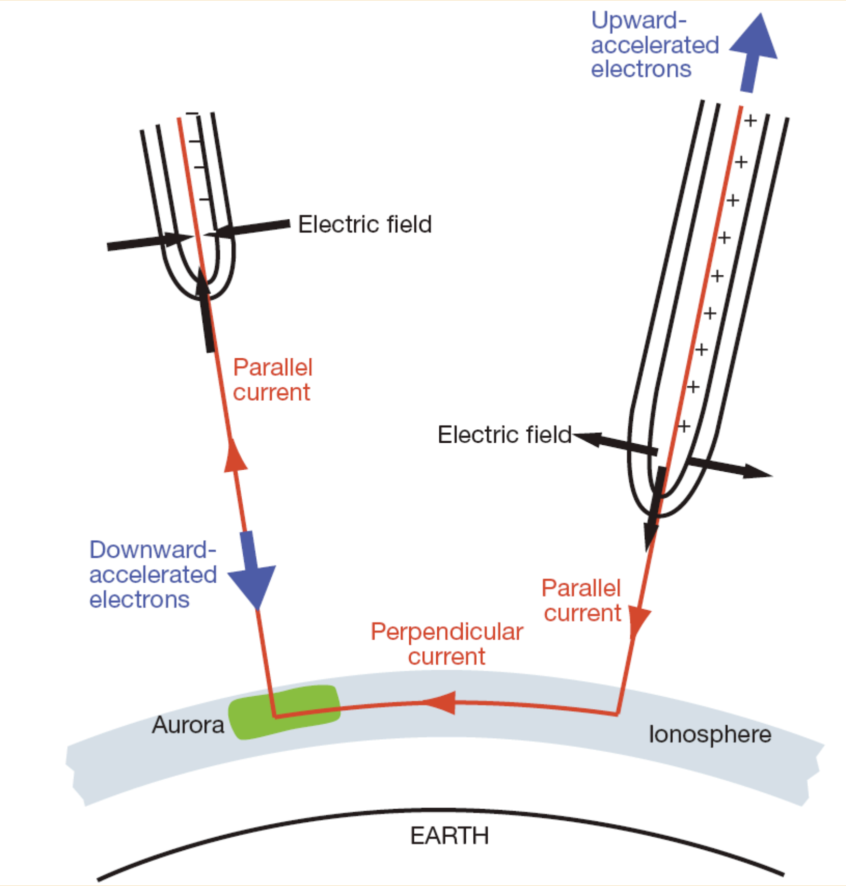
Magnetosphere-Ionosphere Equilibrium Near the Earth

- A simple equilibrium has a total ion number density comprising:
  - constant magnetospheric contribution ($n_M \sim 1\,\mathrm{cm}^{-3}$)
  - exponentially decreasing ionospheric contribution ($n_I \sim 10^4 - 10^6\,\mathrm{cm}^{-3}$, scale height $\sim 100 - 200$ km)



- If $n = n_I + n_M$ and **B** is dipolar, B/n has a peak at a few thousand km altitude
  - below B/n peak: $B/n \sim \exp(+r/h)$
  - above B/n peak: $B/n \sim 1/r^3$

Upward and Downward Currents and the Auroral Acceleration Region



[Marklund *et al.*, *Nature*, 2001.]

- Upward Current:
  - ❍ magnetospheric electrons precepitated ⇒ visible aurora
- Downward Current:
  - ❍ ionospheric electrons evacuated to magnetosphere
- Current-Voltage (energy) relations for upward and downward currents?

Gyrotropic Electron Vlasov Equation: $f(s, v_\parallel, v_\perp, t)$

- Retains information of ionospheric and magnetospheric electron populations

$$\frac{\partial f}{\partial t} + v_\parallel \frac{\partial f}{\partial s} - \left(\frac{eE_\parallel}{m_e} + \frac{v_\perp^2}{2B} \cdot \frac{\partial B}{\partial s} \right) \frac{\partial f}{\partial v_\parallel} + \frac{v_\parallel v_\perp}{2B} \cdot \frac{\partial B}{\partial s} \cdot \frac{\partial f}{\partial v_\perp} = 0$$

$s =$ field-aligned coordinate.

- For a steady current $\mathbf{E} = -\boldsymbol{\nabla}\phi \Rightarrow E_\parallel = -\partial\phi/\partial s$. Constants of motion are
  - total energy: $W = \frac{1}{2} m_e v^2 - e\phi$
  - magnetic moment: $\mu = m_e v_\perp^2 / 2B$

- Solve for $f(s, v_\parallel, v_\perp)$ with $v_\parallel(W, \mu),\ v_\perp(W, \mu)$
  - Liouville's Theorem (f is constant along an electron trajectory)
  - $n_e = n_i \ \Rightarrow \ \phi(s)$ and

$$j_\parallel = -e \int v_\parallel f \mathrm{d}^3 v$$

- Solution relates current flow to potential drop along the field line

Upward Current (downgoing electrons)

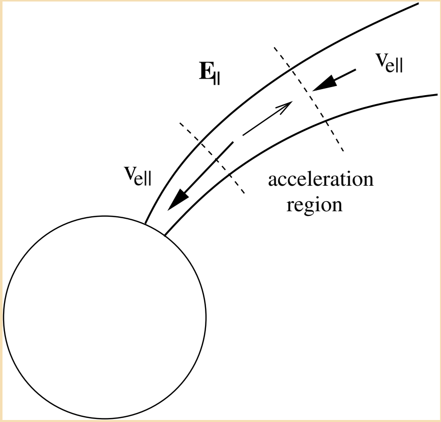
- Assuming a potential drop ϕ_m: map magnetospheric electrons down to the ionosphere using W and μ to identify the loss cone.

- Calculate $j_\parallel$ at the ionosphere in terms of ϕ_m:

$$j_\parallel \approx n_0 e \sqrt{\frac{kT}{2\pi m_e}} \left(1 - \frac{e\phi_m}{kT}\right)$$

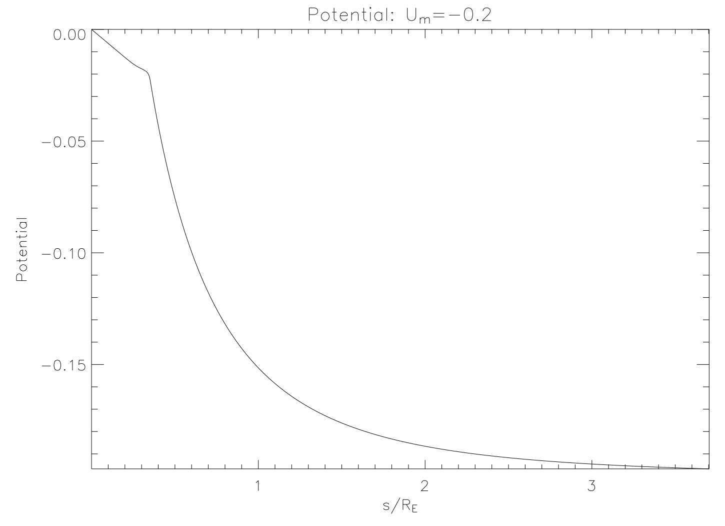
The “Knight” relation [*Planet. Space Sci.*, 1973].

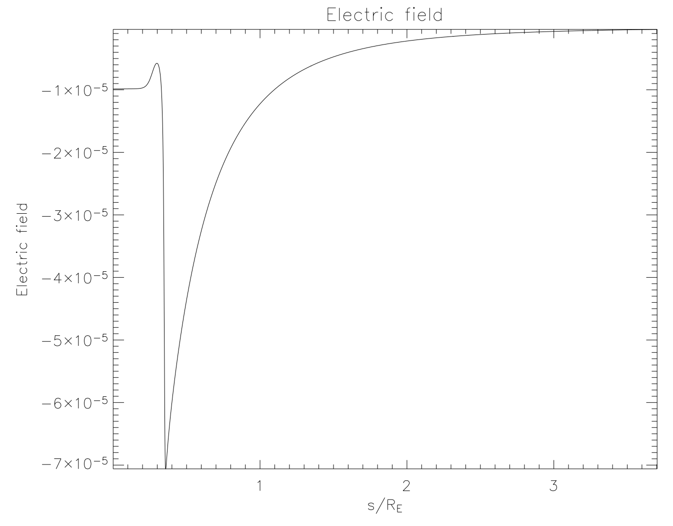
  - Useful for interpreting data (electron energies)
  - Good for incorporating in global modelels
  - Knight’s solution says little about quasi-neutrality or $\phi(s)$ variation
  - $E_\parallel$ needed to overcome mirror force
  - Where does acceleration occur?



Upward Current Quasi-Neutral Solution

- Quasi-neutral solution for previous B/n variation gives [*Cran-McGreehin*, 2006]





- Plots of normalized potential and $E_\parallel$ along **B**. (Ionosphere at $s = 0$)
- Below B/n peak, ambipolar $E_\parallel$ traps ionospheric electrons
- Above B/n peak, $E_\parallel$
  - helps precitating electrons overcome the mirror force
  - adjust mirroring magnetospheric electrons to maintain quasi-neutrality

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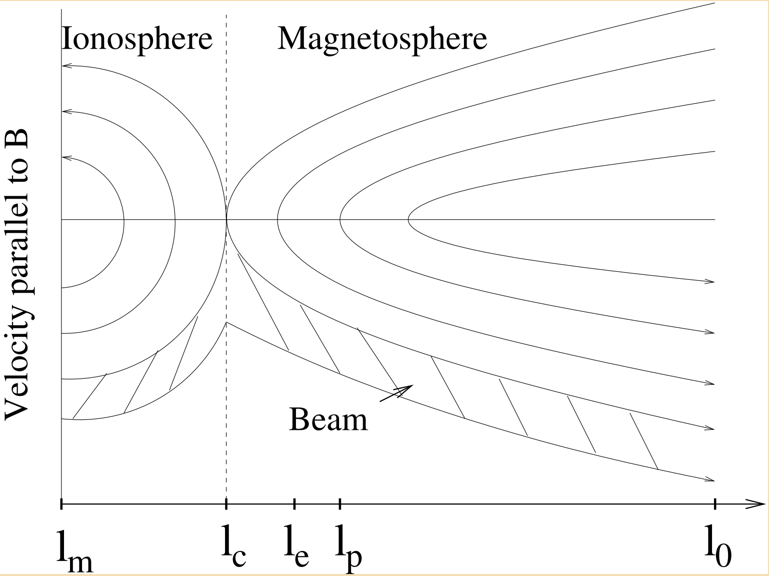
Downward Current (upgoing electrons)

- Some ionospheric electrons are accelerated upward into the magnetosphere

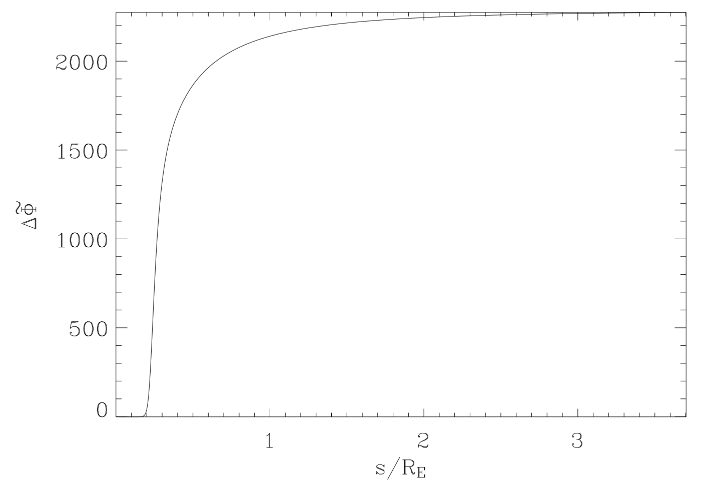


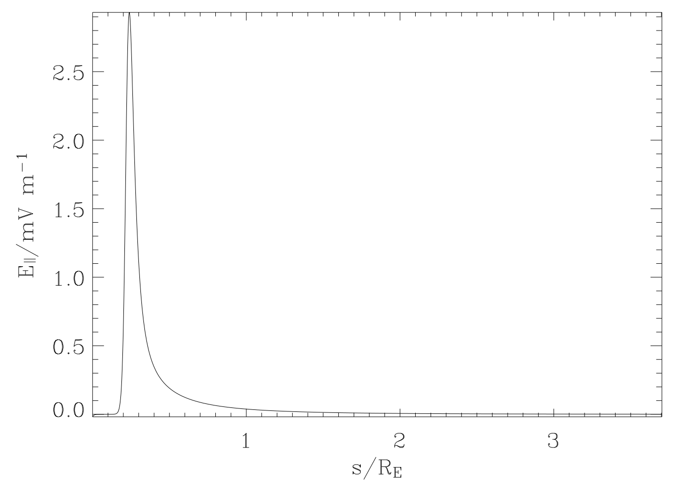
[*Cran-McGreehin and Wright*, JGR, 2005a,b]

- Field-aligned coordinate is ℓ. Important locations are
  - ❍ ℓ_p: location of the B/n peak
  - ❍ ℓ_c: ionospheric electron trapping point/beam emergence height

Downward Current Quasi-Neutral Solution

- Quasi-neutral solution for previous B/n variation gives [*Cran-McGreehin and Wright*, JGR, 2005a,b]





- Plots of normalized potential and $E_\parallel$ along **B**. (Ionosphere at $s = 0$)
- Electron acceleration is centred around the B/n peak
- Ionospheric $j_\parallel$ of $\sim \mu\mathrm{Am}^{-2}$ correspond to potential drops along **B** of ∼keV

Analytical Current-Voltage Relation (Downward Current)

- The potential drop (ϕ_m) depends upon $j_{\|}$ and n at the B/n peak, as well as the magnetospheric electron temperature (T) [*Cran-McGreehin and Wright*, JGR, 2005b]
  - If $j_{\|p}^2 m_e / 2kTn_p^2e^2 < 1.7$ then

$$-\phi_m \approx \frac{3}{2}\left(\frac{j_{\|p}^2 m_e^{1/2} kT}{n_p e^{5/2}}\right)^{\frac{2}{3}} + \frac{1}{2}\left(\frac{j_{\|p}^4 m_e^2 kT}{n_p^4 e^7}\right)^{\frac{1}{3}} + \frac{j_{\|p}^2 m_e}{6n_p^2 e^3}$$

  - If $j_{\|p}^2 m_e / 2kTn_p^2e^2 > 1.7$ then

$$-\phi_m \approx \frac{kT}{e}\ln\left(\frac{j_{\|p}^2 m_e}{n_p^2 e^2 kT}\right) + \frac{j_{\|p}^2 m_e}{2n_p^2 e^3} + \frac{kT}{e}$$

- Approximations accurate to at least 5%
- May only need one or two terms

Conclusion and Summary

- Field aligned currents
  - are an integral part of the magnetosphere
  - arise from both large scale driving and internal particle drifts
  - carried mainly by electrons (accelerated by $E_\parallel$)

- Downgoing electrons excite the aurora
- Details of $\phi(s)$ and $E_\parallel(s)$ are different for upward and downward currents (but B/n peak location is important)
- Analytical Current-Voltage relations available

Future Work

- Allow ions to move and modify background density
- Address time-dependence
- Combine large and small scale physics of acceleration region (FAST)
- Other acceleration mechanisms (wave-particle interactions?)
- Interpret with governing equations