Magnetoseismology for the inner magnetosphere

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Outline

• Introduction
• Techniques
• Examples
• Summary
Magnetospheric seismology
(Magnetoseismology)

- Extraction of information from ULF waves to probe the magnetosphere
- Two wave modes
  - Shear (Alfvén) waves
  - Compressional (fast mode) waves
- Two approaches
  - Normal mode
  - Travel time
- Long history (Alfvén waves)
  - Obayashi and Jacobs [1958]
- Improved measurement and modeling techniques make “Magnetoseismology” relevant
  - Peter Chi [2001]: Fall AGU Meeting

\[ \rho = \rho_s \exp(-R/H_0) \]
Applications

- **Inferring field line mass distribution**
  - Multiple harmonics observed from spacecraft
  - Better density models from single-harmonic measurements on the ground
  - Physics of forces acting on ions
- **Getting information on heavy ions**
  - Comparison with electron density measurements
  - Global ion transport and its dependence on geomagnetic activity
- **Monitoring global mass distribution**
  - Ground magnetometer arrays
  - Plasmapause location and its dependence on the solar wind and geomagnetic activity
Comparison with other seismology

- **Sun and Solid Earth**
  - Steady background medium
  - High-$Q$ resonances
  - Many spectral lines

- **Magnetosphere**
  - Variable background medium
  - Low-$Q$ resonances
  - Small number of observable spectral lines

http://soi.stanford.edu/results/heliowhat.html
MHD wave equation for a cold plasma

- Shear waves
  - Alfvén mode
- Compressional waves
  - Fast mode
- Mode coupling
  - Field line resonance

\[
\rho_0 \frac{\partial \mathbf{v}}{\partial t} = \frac{1}{c} \left( \mathbf{j} \times \mathbf{B}_0 \right)
\]

\[
\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{b}}{\partial t}
\]

\[
\nabla \times \mathbf{b} = -\frac{4\pi}{c} \mathbf{j}
\]

\[
\mathbf{E} = -\frac{1}{c} \mathbf{v} \times \mathbf{B}_0
\]

*Kivelson and Russell [1995]*
Magnetospheric normal mode: 
Standing Alfvén waves

Poloidal mode
- \( E_{\text{azimuthal}} \)
- \( B_{\text{radial}} \)

Toroidal mode
- \( E_{\text{radial}} \)
- \( B_{\text{azimuthal}} \)

Fundamental harmonic \((n = 1)\)

Second harmonic \((n = 2)\)

*Sugiura and Wilson* [1964]
Properties of the inner magnetosphere

- Magnetic field
  - Rigid compared to the outer magnetosphere
  - Numerical models (e.g., Tyganenko)

- Boundary conditions
  - Perfect reflection at the ionosphere a good assumption

- Mass distribution
  - Varies significantly with time and position
  - Functional form not known along field line
Theoretical models of field line distribution of plasma

- Diffusive equilibrium
  - Plasmasphere
  - $\sim R^{-1}$ near the equator

- Collisionless distribution
  - Plasmatrough
  - $\sim R^{-4}$ near the equator
  - Has been popular in the ULF waves community

\[ \frac{n_e}{n_0(\text{eq})} \]

Angerami and Carpenter [1964]
Inferring field line mass distribution

- The frequency of standing waves depends on the spatial mode structure and mass distribution.
  - For example, odd mode (e.g., fundamental mode) is more sensitive to the equatorial mass than even mode (e.g., second harmonic)

- More observable harmonics means more density model parameters (inversion).
  - $N < 10$, realistically, not quite like helioseismology

- Spacecraft measurements are better suited than ground measurements.
  - Frequently yield several harmonics
Toroidal waves at geosynchronous orbit

Takahashi and Denton [2006a]
Statistics of normalized frequency

• Spacing between harmonics
  – Fundamental-second:
    • 0.29-0.32, depends on LT
  – Higher harmonics:
    • ~0.37, varies little

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<th>$f_1/f_3$</th>
<th>$f_2/f_3$</th>
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<td>1.37</td>
<td>1.74</td>
</tr>
</tbody>
</table>

Takahashi and Denton [2006a]
Standing Alfvén wave equation for realistic magnetospheric fields

- Developed by Singer et al. [1981].
- Solved for toroidal harmonics
- Inversion:
  - Parameters of the density models are adjusted so that the observed frequencies match the theoretical frequencies

MHD wave equation:
\[
\frac{\partial^2 (\mathbf{B}_0 \times s)}{\partial t^2} = \nabla \times \nabla \times (\mathbf{B}_0 \times s)
\]

For a given model field \( \mathbf{B}_0 \):
\[
\mu_0 \rho \frac{\partial^2 (s_{\alpha} / h_{\alpha})}{\partial t^2} = \frac{1}{h_{\alpha}^2} \left\{ h_{\alpha}^2 \mathbf{B}_0 \cdot \nabla \left[ h_{\alpha} \mathbf{B}_0 \cdot \nabla (s_{\alpha} / h_{\alpha}) \right] \right\}
\]
Model for mass density variation along field line

Denton et al. [2004]

\[ \log_{10} \rho = c_0 + c_2 \tau^2 + c_4 \tau^4 + c_6 \tau^6 + \ldots \]

\[ \tau \equiv \int_{Eq}^{p} \frac{ds}{V_A} / \int_{Eq}^{N} \frac{ds}{V_A} \]

= 1, Foot point, North
= 0, Equator
= -1 Foot point, South
Density modeling results

- Weaker-than-expected $R$ dependence
  - Closer to $R^{-1}$ (diffusive) than to $R^{-4}$ (collisionless) distribution, although most samples come from the plasmatrough
  - Not far from Polar results for plasmatrough electrons ($\sim R^{-1.7}$) [Goldstein et al., 2001]
- Equatorial maximum in the afternoon
  - Not reported for electrons
  - Equatorial concentration of heavy ions?
  - Potential well at the equator due to rotation?

*Takahashi and Denton* [2006a]
Ion transport within the magnetosphere

- Magnetoseismology provides information on the total ion mass density

Roberts et al. [1987]
Oxygen Torus

- Field line resonance frequency depends on the total mass density
- Plasmapause location depends on particle species
Estimating average ion mass: CRRES results

\[ \rho = n_e m_e + \sum_i n_im_i + n_e m_e \]
\[ \equiv \sum_i n_im_i \]
\[ \equiv n_e M \]

- \( \rho \): Mass density estimated from toroidal frequency, assuming \( R^{-0.5} \) density variation along field line
- \( n_e \): Electron density determined from plasma wave spectra
- \( M \): Average ion mass

Takahashi et al., [2006b]
Inferred average ion mass
Average ion mass: 
Plasmasphere and plasmatrough

- $M$ depends on electron density:
  - High ($> 2$ amu) when $n_e$ is low (plasmatrough)
  - Low ($< 2$ amu) when $n_e$ is high (plasmasphere)

- If [H+, O+] plasma
  - 13% O+ in the plasmatrough

Takahashi et al. [2006b]
Cold ions in the plasmasheet: GEOTAIL observations

Seki et al. [2003]  Hirahara et al. [2004]
Average ion mass:
Dependence on geomagnetic activity

<table>
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<th>Dst (nT)</th>
<th>M (amu)</th>
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<tr>
<td>0</td>
<td>2.3</td>
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<td>-20</td>
<td>2.6</td>
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<td>-40</td>
<td>3.3</td>
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<td>-60</td>
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Heavy ions

• Present both inside and outside the plasmasphere
• Increases with geomagnetic activity ($D_{st}$)
• Consistent with GEOTAIL studies of the plasmasheet and dayside outer magnetosphere
• Cold ion transport processes yet to be identified
Cross phase technique: How it works

• Based on the concept of field line resonance
• Uses latitudinal pairs separated by ~100 km
• Cross phase shows a peak at the resonance frequency of the field line at the midpoint of the stations.
  – Clearer signature than amplitude ratio or spectral peak in the single-station power spectra

Waters et al. [1991]
Cross phase technique:
Tracking the temporal variation of density

2001 April 18 (Day 108) 1531 UT

IMAGE EUV plasmapause:
courtesy of J. Goldstein
FLRs are always present on the dayside.

With dense latitudinal ground magnetometer arrays we can monitor the density structure near the plasmapause as a function of time.

[Menk al., 2004]
Short-time scale (1 hour) density variation

- Possible causes
  - Enhanced convection electric field
  - $\textbf{E} \times \textbf{B}$ drift
  - Redistribution of $\text{O}^+$ ions near the plasmapause

[Menk et al., 2004]
Solar cycle variation

- Mass density variation at $L \sim 7$
  - Changes by a factor of $\sim 10$
  - Comparable to changes at the topside ionosphere

[Lean, 1997]  
[Takahashi et al., 2002]
Fast mode waves: cavity mode resonance

- Pi2 pulsations (nightside)
- Si/Sc-associated pulsations (dayside)
- Strongly damped
- Boundaries
  - Magnetopause
  - Plasmapause

*Denton et al.* [2002]
Low-latitude Pi2: plasmaspheric normal mode

Takahashi et al. [2003]
Pi2 frequency: Dependence on Lpp

\[ f_{Pi2} = \frac{V_A}{2R_E (L_{pp} - 1)} \]

Takahashi et al. [2003]
Pi2 frequency: Dependence on local time

\[ f_{\text{dusk}} < f_{\text{midnight}} \sim f_{\text{dawn}} \]

Takahashi and Liou [2004]
Fast mode/shear mode waves: travel time seismology

[Chi et al., 2005]
Travel time seismology

Density model:
Power-law variation with $L$ with 5 free parameters

$$t_{Tamao} = \int_{l_1} ds \frac{ds}{v_f(r)} + \int_{l_2} ds \frac{ds}{v_A(r)}$$

$$\chi^2 = \sum_i \left( \frac{t_{obs,i} - t_0 - t_{Tamao,i}}{\sigma_i} \right)^2$$

[Chi et al., 2005]
Summary

• Magnetospheric seismology is a unique technique for probing the magnetosphere
  – Spatial and temporal variation of mass distribution
  – Total mass density (heavy ion contribution to the magnetospheric plasma)
• Various approaches
  – Spacecraft and ground observations
  – Fast mode and shear mode
  – Normal mode and travel time
• Recent results
  – Storm time ion transport
  – Plasmapause dynamics
• Future directions
  – More magnetometers on the ground
  – Improvement in magnetic field model and inversion techniques
References


Singer, H. J., D. J. Southwood, R. J. Walker, and M. G. Kivelson (1981), Alfvén wave resonances in a realistic magnetospheric magnetic field geometry, *J. Geophys. Res.*, 86(A6), 4589–4596.


